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A finite element analysis program for reverberant spaces /

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A

Finite Element Analysis Program

for

Reverberant Spaces

By

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A Finite Element Analysis Program for Reverberant Spaces

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CERTIFICATE OF APPROVAL

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LIST OF SYMBOLS

A = coefficient of orthogonal series function

Q = coefficient of orthogonalized source function

$Q_\circ$ = source strength

T = temperature

V = volume of a rectangular enclosure

c = speed of sound

$f_n = \omega/2\pi$ = natural frequency

f_{dr} = driving frequency

$k = \omega/c$ = wave number = square root of eigenvalue

l,m,n = indicies of the mode shape

p = acoustic pressure

q = source input

r = a point in space

$r_\circ$ = location of the point source

t = time

x_L, y_L, z_L = dimensions of the room

β = wall absorption

ϕ = velocity potential

ψ = orthogonalized function

ω_n = angular frequency

Δ = second order differential operator "del squared"

LIST OF CONSTANTS

The following constants were used in this research:

P_{ref} = 0.00002 Newtons per square meter

V_{ref} of source = 0.113 meters per second

radius of source = .07 meters

Temperature = 24.5° Centigrade

Air Density = 1.21 kilograms per cubic meter

Absorption = 0.005 to 0.050 where 0 = totally reflective and 1 = totally absorptive

ABSTRACT

A finite element analysis program has been developed to analyze the spatial distribution of pressure in irregularly shaped reverberant enclosures driven by a point source. The program uses a matrix form of the Helmholtz Equation modified with a driving source term. Soft wall boundary conditions are introduced with Galerkin's method to yield a complex symmetric matrix.

The results compare well with the closed form solution for a rectangular room with a point source and homogeneous absorption material on all walls over the low range of frequencies and for several absorption coefficients. However, at higher frequencies corresponding to higher eigenvalues, the low mesh density introduces error similar to an increase in stiffness. Irregular shapes have also been studied. Effective use of the program for spatial distribution and frequency response analysis for forced vibration requires appropriate finite element mesh density, adequate computer resources and computer graphics for interpretation of results.

I. INTRODUCTION

The purpose of this study is to develop a finite element analysis method for the acoustic response in enclosures with wall damping and point sources. By comparing the finite element solution with an orthogonal function series solution for a rectangular enclosure, which is known to agree with physical experimental results, confidence in the validity of the method can be gained. The method's applicability to irregularly shaped enclosures is also investigated.

I. A. BACKGROUND

Determination of the acoustic pressure response of an enclosure with wall damping and point sources has been of interest to architectural and noise engineers for some time. For example, the design of rooms, large auditoriums, airplane and automobile interiors, work areas, and duct and piping would benefit from this analysis. Exact solutions are impossible to formulate due to the large variety of unusual geometry and boundary conditions, as well as limitations in solution methods, leaving only approximate solution techniques. Solutions do exist for regular shaped enclosures, such as cylindrical and rectangular shapes, as an infinite series of orthogonal functions, a closed form method for solving differential equations. Separation of

Variables is used to develop the homogeneous solution to the differential equation, and expansion of a Fourier's style orthogonal function series solution is used to solve the differential equation for a rectangular room with one source and the same absorption on all walls.

The development of the closed form solution of the wave equation as it pertained to standing waves of sound in a rectangular enclosure with a point source and wall damping, which is presented in this paper, first appeared in a 1936 text [1] by Morse. Five years later, Feshbach [2-3] proposed a perturbation method for the solution of irregular shapes, slightly distorted from a shape with a closed form solution. Lyons [4] studied noise reduction of rectangular enclosures with one flexible wall, particularly in relation to sound transmission. Kuttroff provides much of the same development with more emphasis on geometric effects of the room, such as shape and wall absorption qualities in his 1973 text [5]. He also analytically studied reverberation in enclosures containing scattering and absorptive elements in references [6-8], and lately has turned his attention to propagation of sound in flat enclosures [9].

However, the closed form solution is weak in that it is only applicable to regular shaped enclosures. A simplified version of this solution, in which the room dimensions are equal, could permit different absorption on each wall, but designers rarely encounter the ideal situations demanded by these solutions.

Other techniques have been devised to study enclosures, such as tracing the sound waves as vectors, or by using an approximation method, as used by Perry [10], who studied the noise levels due to several sources of machinery in large enclosures. Jennequin [11] and Shuku [12] used the finite difference method to evaluate the natural frequencies and mode shapes of the passenger space in a car.

The finite element technique provides a numerical approach to this problem free from the geometrical constraints of the normal Morse approach. In this technique, the enclosure is considered to be a conglomeration of smaller blocks or elements over which the differential equation is imposed through interpolation functions. Galerkin's technique of calculus of variations is used to develop the finite element solution. There are several advantages to this method. The enclosure need not necessarily be rectangular, and the properties of each element can vary from one another, permitting much more diversity of application of the solution obtained under this technique. In addition, a standard format can accommodate a large variety of problems, while variables can be changed easily to facilitate a search for optimum design. Response can be determined in a room where several sources are emitting sound of the same frequencies or strengths. Natural frequencies, which can provide the designer with information about the fundamental frequencies and mode shapes, can also be computed by this technique. The finite element equations developed in this study predict the actual room response when the air is driven at a given frequency, as opposed to finding mode shapes of natural frequencies.

Since about 1940, the finite element method has evolved as a solution technique for a variety of problems. Initially, it was developed as a method to solve problems in elastic, solid structures. It was immediately applied to structural analysis, hydraulic conduit flows, and electronic circuits. The calculus of variations approach to the finite element method has been applied to solid mechanics, fluid dynamics, heat transfer, creep theory, lubrication, and acoustic analysis.

Gladwell [13-14] first presented the finite element method for determining the frequencies and mode shapes of acoustic cavities in 1965, using sections built up from rectangular elements. Shuku [15] developed a two dimensional acoustic element to calculate the natural frequencies and modes in irregularly shaped rooms. In 1972, Craggs [16] developed a three dimensional hexahedral element to determine the natural frequencies and mode shapes of complex shaped enclosure, an approach mentioned in this paper. Petyt [17] developed a highly accurate twenty node isoparametric brick element which could be used to represent spaces with curved boundaries. Hertig [18] used annular elements to analyze a solid rocket motor using NASTRAN. Craggs [19] used finite elements to study boundary flexibility and the transmission of sound between enclosures. In 1976, Craggs [20-21] applied a model capable of handling damped acoustic systems to study a muffler. Petyt [22] showed the application of the finite element solution to an acoustic cavity with internal and wall sources and damping. Ling [23] used Galerkin's Method in a two dimensional approach to acoustic flow in ducts. Kung [24-25] is working with three dimensional annular acoustical elements and has applied a modal analysis technique. Bernhard [26] developed a technique of multiplying an original matrix by a shape change parameter to develop a model of different geometries.

I. B. PROBLEM STATEMENT

The orthogonal series solution for a rectangular room with soft walls and a point source had been developed in 1936 and the finite element method is readily applicable to the same situation. This presented the opportunity to study the finite element method against a known solution to compare the results. Furthermore, it provided the opportunity to develop the acoustical finite element equations for a point source, and to develop the numerical implementation of assigning a physical constant to the sides of an element, rather than the nodes. This finite element model can be used to predict the frequency reponse of any enclosure with soft walls, regardless of shape or size, at any frequency, regardless of source strength, pitch, or number of sources.

The solution techniques developed in this paper are formulated upon the following assumptions [27-29]:

1. The Equation of Continuity, borrowed from physics, is the springboard of much of the development.

2. Newton's Equation of Force is presented in the development of the differential equation.

3. The fluid has elastic properties.

4. The process is adiabatic.

5. Local density changes are small.

6. The displacement and velocity of the fluid particle are small.

7. Driving frequencies are produced from point sources.

8. Absorption coefficients are small.

9. Finite element bricks must not be distorted too far from square. (Avoid angles greater than 180° and long, thin elements.)

I. C. THESIS ORGANIZATION

This paper is divided into five sections. The first section contains background information about why this problem is of interest, and the work that has been done in this area. The second and third sections contain the mathematical formulation of the equations from physics through the development of both a traditional infinite orthogonal function series and the finite element model. Each section contains numerical implementation of the solution techniques on a digital computer, along with the complications and requirements of each method. In the fourth section, results of comparing the two solutions are presented, including data from actual computer runs to prove the general agreement of both solutions, as well as the agreement at driving frequencies near natural frequencies to show the validity of the finite element solution at these critical frequencies. The validity of the solution at varying wall absorptions is also considered. Finally, conclusions are drawn as to the validity of the finite element solution, its power, and its limitations. Recommendations for further research and deveplopment are also presented.

II. MATHEMATICAL DEVELOPMENT OF THE SERIES SOLUTION

II. A. DEVELOPMENT OF THE WAVE EQUATION [1,5,28]

The Equation of Continuity, which states that a medium does not separate, is:

$$\rho_\circ\ \partial u/\partial x = -\ \partial\rho/\partial t + \rho_\circ\ q \qquad \text{II.1}$$

where $\rho_\circ$ = initial density

ρ = density

u = a direction

t = time

q = source input

Breaking the density into a static term ($\rho_\circ$) and one that varies with time and space (δ), then ρ can be written as

$$\rho = \rho_\circ + \delta\ . \qquad \text{II.2}$$

Then the term that varies (δ) is a function of the acoustic pressure (p) [30] and is given by

$$p = c^2\delta \quad \text{or} \quad \delta = p\ /\ c^2\ . \qquad \text{II.3}$$

Taking the derivative of the density in Equation II.2 with respect to time and substituting in the acoustic pressure term from Equation II.3 yields

$$\partial\rho/\partial t = 0 + \partial\delta/\partial t = 1/c^2\ \partial p/\partial t\ . \qquad \text{II.4}$$

Replacing this expression back into Equation II.1, results in the following expression of the Equation of Continuity in terms of acoustic pressure:

$$\rho_\circ\ \partial u/\partial x = -\ 1/c^2\ \partial p\ /\ \partial t + \rho_\circ\ q\ . \qquad \text{II.5}$$

Newton's Equation of Force states that force is equal to mass times acceleration. For a differential pressure in one direction with constant cross-sectional area, this equation can be written as

$$\partial p/\partial x = - \rho_\circ \, \partial u/\partial t \ . \qquad \text{II.6}$$

Taking the derivative of this with respect to x yields

$$\partial^2 p/\partial x^2 = - \rho_\circ \, \partial^2 u/(\partial t \partial x) \ . \qquad \text{II.7}$$

The derivative of Equation II.5 with respect to time yields

$$\rho_\circ \, \partial^2 u/(\partial x \partial t) = - 1/c^2 \ \partial^2 p/\partial t^2 + \rho_\circ \, \partial q/\partial t \ . \qquad \text{II.8}$$

Equating the right hand side of Newton's Equation of Force, Equation II.7, with the left hand side of the Continuity Equation II.8, the following one dimensional form of the Wave Equation is obtained:

$$\partial^2 p/\partial x^2 = 1/c^2 \ \partial^2 p/\partial t^2 - \rho_\circ \, \partial q/\partial t \ . \qquad \text{II.9}$$

To eliminate directionality, a velocity potential (ϕ) can be defined as

$$u = - \partial \phi/\partial x \ , \qquad \text{II.10}$$

where the velocity in the x-direction is given by

$$u = - 1/\rho_\circ \int \partial p/\partial x \ \partial t \ , \qquad \text{II.11}$$

and the acceleration in the x-direction is given by

$$\partial u/\partial t = - 1/\rho_\circ \, \partial p/\delta x \ . \qquad \text{II.12}$$

Incorporating the velocity potential from Equation II.10 into Newton's Equation of Force, Equation II.6, yields

$$\partial p/\partial x = - \rho_\circ \partial u/\partial t = \rho_\circ \, \delta^2 \phi/(\partial x \partial t) \ , \qquad \text{II.13}$$

and integrating this with respect to x, yields acoustic pressure as a function of the velocity potential

$$p = \rho_\circ \, \partial\phi/\partial t \; . \qquad \text{II.14}$$

Taking two derivatives of this with respect to x, and incorporating that result into Equation II.9 yields the Wave Equation in terms of the velocity potential. Thus,

$$\partial^2 p/\partial x^2 = \rho_\circ \, \partial^3\phi/(\partial x^2 \partial t) = \rho_\circ/c^2 \;\; \partial^3\phi/\partial t^3 - \rho_\circ \, \partial q/\partial t \; .$$

or

$$\partial^2\phi/\partial x^2 = 1/c^2 \; \partial^2\phi/\partial t^2 - q \; . \qquad \text{II.15}$$

Assuming simple harmonic motion, then the velocity potential can be expressed as a coefficient times an alternating function

$$\phi(t) = \Phi \; e^{i\omega t} \; . \qquad \text{II.16}$$

This expression in Equation II.14 becomes

$$p = \rho_\circ \, \partial\phi/\partial t = i\omega\rho_\circ \; \Phi e^{i\omega t} = i\omega\rho_\circ \;\; \phi(t) \; , \qquad \text{II.17}$$

and in this case of simple harmonic motion, the velocity potential can be expressed by rearranging this equation as

$$\phi(t) = p/(i\omega\rho_\circ) \; . \qquad \text{II.18}$$

Taking two derivatives of this with respect to x so that it may be incorporated into the right hand side of the Wave Equation II.15 yields

$$\partial^2\phi/\partial x^2 = 1/(i\omega\rho_\circ) \; \partial^2 p/\partial x^2 \; . \qquad \text{II.19}$$

Taking two derivatives with respect to t of Equation II.16, where the velocity potential is a function of time, yields

$$\partial^2\phi/\partial t^2 = -\,\omega^2\Phi e^{i\omega t} = -\,\omega^2 \; \phi(t) = -\,\omega p \; / \; (i\rho_\circ) \; . \qquad \text{II.20}$$

Replacing these two results back into Equation II.15, yields the Wave Equation in terms of acoustic pressure as

$$1/(i\omega\rho_\circ)\ \partial^2 p/\partial x^2 = -\ \omega p/(i\rho_\circ c^2) - q\ ,$$

or
$$\partial^2 p/\partial x^2 + k^2 p = -i\omega\rho_\circ q\ , \qquad \text{II.22}$$

where the wave number (k) is defined as

$$k = \omega/c\ . \qquad \text{II.23}$$

In three dimensions, the Wave Equation for simple harmonic motion in terms of acoustic pressure is given by

$$\Delta p + k^2 p = -i\omega\rho_\circ q \qquad \text{II.24}$$

where Δ is the second order differential operator "del squared" defined as

$$\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2\ .$$

The right hand side of Equation II.24, when set equal to zero, is the Helmholtz Equation. The right hand side is the driving source term.

II. B. SOLUTION OF THE FORCED WAVE EQUATION IN A RECTANGULAR ENCLOSURE BY ORTHOGONAL SERIES [28]

The Forced Wave Equation

$$\Delta p + k^2 p = -i\omega\rho_\circ q \ , \qquad \text{II.25}$$

can be solved using separation of variables for certain simple geometries by assuming a solution that can be represented by an infinite series of weighted orthogonal functions:

$$p = \Sigma_n A_n \psi_n \ , \qquad \text{II.26}$$

where ψ represents the orthogonal functions and A is the weight.

II. B. 1. The Homogeneous Solution

Solving the homogeneous problem with this solution yields

$$\Sigma_n A_n \Delta\psi_n + \Sigma_n k_n^2 A_n \psi_n = \Sigma_n A_n(\Delta\psi_n + k_n^2 \psi_n) = 0 \ . \qquad \text{II.27}$$

In order that the coefficients not be equal to zero, the functions themselves must add to zero

$$\Delta\psi_n + k_n^2 \psi_n = 0 \ ,$$

$$\text{or } \Delta\psi_n = - k_n^2 \psi_n \ . \qquad \text{II.28}$$

In three dimensions, this equation is written

$$\partial^2\psi/\partial x^2 + \partial^2\psi/\partial y^2 + \partial^2\psi/\partial z^2 + k^2\psi = 0 \ . \qquad \text{II.29}$$

The separable solution in this case is

$$\psi = \Sigma_{lmn} A_{lmn} \cos(l\pi x/X_L) \cos(m\pi y/Y_L) \cos(n\pi z/Z_L) \ , \qquad \text{II.30}$$

where $\cos(l\pi x/X_L)$ $\cos(m\pi y/Y_L)$ $\cos(n\pi z/Z_L)$ is the orthogonalized function, X_L, Y_L, Z_L are the room dimensions, and A_{lmn} are constants to be determined by the source and boundary conditions.

II. B. 2. The Eigenvalue Problem

It can be shown that the Eigenvalues for the homogeneous solution are then given by

$$\begin{aligned} k^2 &= k_l^2 + k_m^2 + k_n^2 \\ &= (\pi l/X_L)^2 + (\pi m/Y_L)^2 + (\pi n/Z_L)^2 \\ &= \pi^2 \{ (l/X_L)^2 + (m/Y_L)^2 + (n/Z_L)^2 \} \end{aligned} \qquad \text{II.31}$$

and the corresponding Eigenfunction for any l,m,n is of the form

$$\psi_n = \psi_{lmn} = A_{lmn}\cos(l\pi x/X_L)\cos(m\pi y/Y_L)\cos(n\pi z/Z_L) \ . \qquad \text{II.32}$$

Since angular frequency can be expressed as

$$\omega = 2\pi f_{lmn} \ , \qquad \text{II.32}$$

and $$k = \omega/c \ , \qquad \text{II.33}$$

it follows that the natural frequencies are given by

$$f_{lmn} = (c/2) \ \{(l/x_L)^2 + (m/y_L)^2 + (n/z_L)^2 \}^{1/2} \ . \qquad \text{II.34}$$

II. B. 3. The Particular Solution

Taking two derivatives of the assumed series solution Equation II.26 with respect to space, and using Equation II.28, gives

$$\Delta p = \Sigma_n A_n \Delta\psi_n = \Sigma_n -A_n k_n^2 \psi_n \ . \qquad \text{II.35}$$

Assuming the source is also a simple harmonic generator, the source function can be expanded into a series with the same functions as the homogeneous case. That is

$$q(r_\circ)\ e^{i\omega t} = \Sigma_m Q_m \psi_m(r)\ e^{i\omega t}\ , \tag{II.36}$$

where r is a vector and $r_\circ$ is the location of the point source. Multiplying both sides by the orthogonal function ψ_n and integrating over the volume yields

$$\int_V q(r_\circ)\psi_n e^{iwt}\partial V = \Sigma_m Q_m \int_V \psi_m(r)\ \psi_n(r)\ e^{i\omega t}\partial V\ . \tag{II.37}$$

The integral over the volume of the product of two orthogonal functions together is given by

$$\int_V \psi_n{}^2 \partial V = V\Lambda_n \text{ for } m = n\ , \text{ and}$$

$$= 0 \quad \text{for } m \neq n\ , \tag{II.38}$$

where V is the volume of the room which is

$$V = x_L\ y_L\ z_L\ , \tag{II.39}$$

and Λ_n is to be determined. Using Equation II.38 and solving Equation II.37 for the coefficient Q_n, gives the weights for the expansion of the source in the orthogonal series

$$Q_n = 1/(V\Lambda_n)\ \int_V\ q(r_\circ)\ \psi_n(r)\ \partial V\ . \tag{II.40}$$

Replacing the series representations of Equations II.35 and II.36 for the solution and the source back into the Wave Equation II.25 yields

$$\Sigma_n[-A_n k_r{}^2 \psi_n + (\omega/c)^2\ A_n \psi_n] = -i\omega\rho_\circ\ \Sigma_n Q_n \psi_n\ . \tag{II.41}$$

For this summation to be equivalent, the coefficients for each term must be equivalent and therfore

$$A_n = i\omega\rho_\circ Q_n/(k_n^2 - k^2) \ . \qquad \text{II.42}$$

Substituting Q_n from Equation II.40 in this expression yields

$$A_n = i\omega\rho_\circ/(V\Lambda_n) \int_V q(r_\circ)\ \psi_n(r)\ /(k_n^2 - k^2)\ \ \partial V \ . \qquad \text{II.43}$$

Placing this expression back into $p = \Sigma_n\ A_n\psi_n$ yields

$$p(r) =$$

$$i\omega\rho_\circ/V\ \Sigma_n[\ \int_V\ q(r_\circ)\psi_n(r)\partial V\ ]\ \psi_n(r)\ /[\Lambda_n(k_n^2-k^2)] \ . \qquad \text{II.44}$$

Considering only the case where the source is a point, then

$$q(r_\circ) = Q_\circ\delta(r-r_\circ) = Q_\circ\delta(x-x_\circ, y-y_\circ, z-z_\circ) \ . \qquad \text{II.45}$$

Integrating this in the presence of the orthogonality function gives

$$\int_V\ Q_\circ\delta(r-r_\circ)\psi_n(r)\ \partial V = Q_\circ\ \psi_n(r_\circ) \ . \qquad \text{II.46}$$

Then acoustic pressure for a room with a point source of strength $Q_\circ$ is

$$p(r) = i\omega\rho Q_\circ/V\ \ \Sigma_n\ \{\psi_n(r_\circ)\ \psi_n(r)\}/\{\Lambda_n(k_n^2-k^2)\} \ . \qquad \text{II.47}$$

At this point, attention is focused on Λ_n in the denominator. As shown above

$$\int_V\psi_n^2\partial V = V\Lambda_n \text{ for } m = n ,$$

$$= 0 \quad \text{for } m \neq n , \qquad \text{II.48}$$

$$\psi_n = \psi_{lmn} = (\cos k_l x)(\cos k_m y)(\cos k_n z) \ , \qquad \text{II.49}$$

and $$V = X_L Y_L Z_L \ , \qquad \text{II.50}$$

then $\int_V \psi_n^{\,2} \partial V =$

$$\int_0^{ZL} \int_0^{YL} \int_0^{XL} (\cos^2 k_l x)(\cos^2 k_m y)(\cos^2 k_n z)\ \partial x \partial y \partial z\ . \qquad \text{II.51}$$

Treating the other two terms as constant during the integration of the third term results in

$$\int_0^{XL} (\cos^2 k_l x)\ \partial x = \int_0^{XL} [1/2 + 1/2 \cos\ (2k_l x)\ ] \partial x$$

$$= 1/2\ X_L + 1/(4k_l)\ \sin\ (2k_l x) \simeq 1/2\ X_L \text{ for } l,m,n > 0, \qquad \text{II.52}$$

and $\int_V \psi_n^{\,2}\ \partial V = \int_0^{XL} \partial x = X_L$ for $l,m,n = 0$. II.53

Thus, if letting $\epsilon(n)=1$ for $n=0$, and $\epsilon(n)=2$ for $n>0$, yields

$$\Lambda_n = 1/[\epsilon(l)\ \epsilon(m)\ \epsilon(n)] = 1/E(l,m,n)\ , \qquad \text{II.54}$$

and $$p(r) = i\omega\rho Q_o/V\ \Sigma_n\{E(n)\psi_n(r)\psi_n(r_o)\}/(k_n^{\,2}-k^2)\ , \qquad \text{II.55}$$

where $$E(l,m,n) = E(n) = \epsilon(l)\ \epsilon(m)\ \epsilon(n)\ . \qquad \text{II.56}$$

In the case of soft walls, k is complex, where the real part of k is the wave number for hard walls (ω/c), and the imaginary part contains the damping coefficient (δ_n). Thus,

$$k = (\omega/c) + i\ \delta_n\ , \qquad \text{II.57}$$

and $$k^2 = (\omega/c)^2 + 2i(\omega/c)\delta_n + \delta_n^{\,2}\ . \qquad \text{II.58}$$

If the damping is small, δ_n^2 can be neglected. Letting $k = \omega/c$ (as for hard walls), the denominator becomes

$$k^2 - k_n^{\,2} + 2ik\delta_n\ . \qquad \text{II.59}$$

Incorporating the boundary condition [22]

$$\partial p/\partial \nu + (i\omega\beta/c)\ p = 0\ , \qquad \text{II.60}$$

yields [5]

$$\delta_n = [\ \epsilon(l)/X_L + \epsilon(m)/Y_L + \epsilon(n)/Z_L\ ]\beta\ . \qquad \text{II.61}$$

Finally, the orthogonal series closed form solution may be expressed as

$$p(r,t) =$$

$$(i\omega\rho Q_\circ/V)\ \Sigma_n\{\ E_n\ \psi_n(r_1)\ \psi_n(r_0)\ e^{-i\omega t}\}/\{k^2-k_N{}^2+2ik\delta_n\}\ . \qquad \text{II.62}$$

where $i = \sqrt{-1}$, II.63

$\omega = 2\pi f_{dr}$, II.64

f_{dr} = driving frequency , II.65

ρ = air density , II.66

$Q_\circ$ = source strength , II.67

V = room volume = $X_L Y_L Z_L$, II.68

$E(l,m,n) = E(n) = \epsilon(l)\ \epsilon(m)\ \epsilon(n)$ II , II.69

$\epsilon(n)=1$ for $n=0$, and $\epsilon(n)=2$ for $n>0$, II.70

$\psi_n = \psi_{lmn} = (\cos k_l x)(\cos k_m y)(\cos k_n z)$, II.71

$k_l,\ k_m,\ k_n = l\pi/X_L,\ m\pi/Y_L,\ n\pi/Z_L$, II.72

$k = \omega/c$, II.73

$k_N = \omega_N/c$, II.74

$\omega_N = 2\pi f_N$, II.75

f_N = the n th natural frequency , II.76

and $\delta_n = [\ \epsilon(l)/X_L + \epsilon(m)/Y_L + \epsilon(n)/Z_L\ ]\beta$. II.77

II. C. COMPUTER IMPLEMENTATION OF THE ORTHOGONAL SERIES SOLUTION

II. C. 1. Compute and Order the Natural Frequencies

The orthogonal series solution

$$p(r,t) =$$

$$(i\omega Q_\circ/V)\ \Sigma_n\{\ E_n\ \psi_n(r_1)\ \psi_n(r_0)\ e^{-i\omega t}\}/\{k^2-k_N{}^2+2ik\delta_n\} \qquad \text{II.78}$$

requires an infinite number of terms to converge to a solution. Obviously, then, another approach is necessary. A complication arises in that the first terms of the series are not necessarily the most significant in the contribution to the sum. In the denominator of the orthogonal series solution appears the term $k^2-k_N{}^2$ which has the most significant effect on the size of the terms in the series. As this term shrinks, the contribution becomes larger. Hence, the terms which matter most in the formation of the sum are the terms close to the driving frequency. But even among these terms, the contribution varies dramatically from term to term, since each term represents an eigenfunction which can vary greatly across the room, especially for the lower modal shapes. For instance, a point may be located in space at a place where it is at the node for one mode shape, but at the peak of another mode shape. Therefore, one cannot simply begin and truncate the series at the first low term, since the next term in the series may contribute significantly to the sum.

Ochs [31] has shown that at low frequencies, it is sufficient to include only the 100 terms corresponding to natural frequencies nearest to the driving frequency. Therefore, computer implementation for the orthogonal series requires knowledge of the natural frequencies and their order.

The natural frequencies for a room of dimensions x_L, y_L, z_L are given by the equation

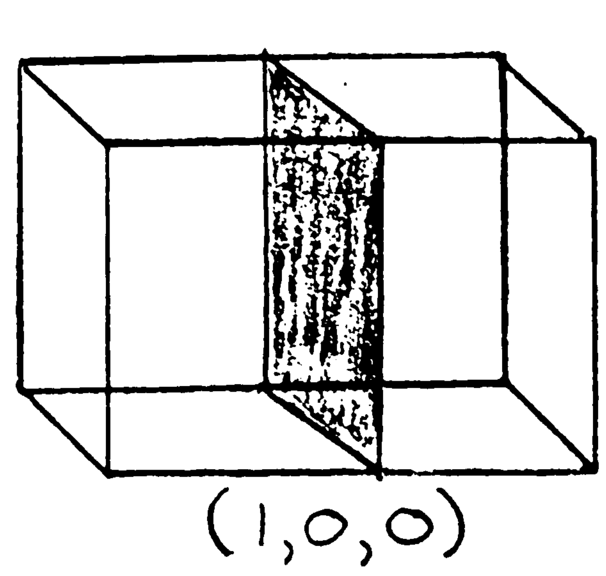
$$f_{l,m,n} = c/2 \;\{ (l/x_L)^2 + (m/y_L)^2 + (n/z_L)^2 \}^{1/2} , \qquad \text{II.79}$$

where c is the speed of sound in meters per second, which is

$$c = 331.6 \;\{ (273 + T)/273 \}^{1/2} , \qquad \text{II.80}$$

and T is the temperature in Centigrade

These frequencies are computed by chosing values for l, m, and n for a given room. As shown in Illustration 1, for (l,m,n) of (1,0,0), the frequency will correspond to a mode shape where there is one nodal plane which lies at the center of the x-axis in the y-z plane. A nodal plane or surface is the surface of a natural frequency response where acoustic pressure is low. Similarly, for (l,m,n) of (2,1,0), the frequency will correspond to a mode shape with three nodal planes: one at the center of the y-axis in the x-z plane, and two nodal planes on the x-axis, located at the first and third quarter points, in the y-z plane.



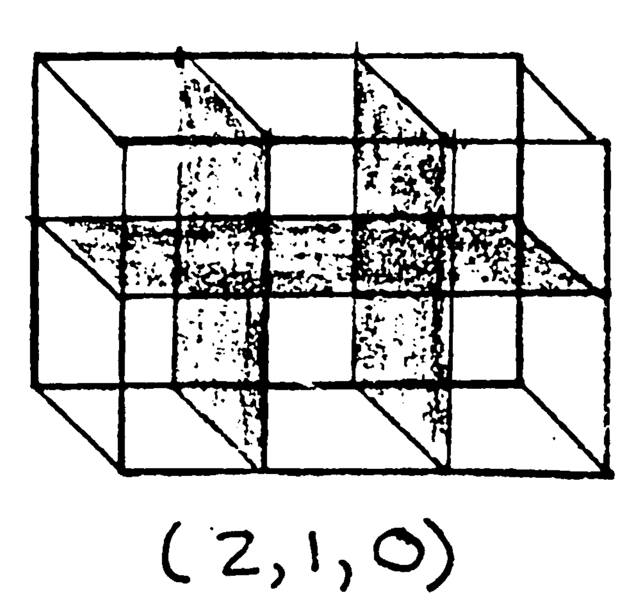


Illustration 1. Mode Shapes (1,0,0) and (2,1,0)

To analyze the sum, the maximum driving frequency believed appropriate must first be selected. Next, limits for the indices (l,m,n) are chosen. The maximum values of (l,m,n) must be chosen such that fifty modes beyond the natural frequency nearest the highest anticipated driving frequency can be constructed. However, if the limits of (l,m,n) are set too high, the computer must compute, order, and store values which will never be used. So care must be taken to ensure that all the natural frequencies which might contribute to the sum for the highest analyzed driving frequency are included. To do this, f_{lmn} is solved for the index l nearest the highest driving frequency in this range, with the other two indicies (m and n) both set to 0. This process is repeated for m and n:

$$l_{max} = (2 f_{dr} X_L)/ c + (1 \text{ or } 2 \text{ or } 3 \ldots), \qquad \text{II.81}$$

$$m_{max} = (2 f_{dr} Y_L)/ c + (1 \text{ or } 2 \text{ or } 3 \ldots), \qquad \text{II.82}$$

and

$$n_{max} = (2 f_{dr} Z_L)/ c + (1 \text{ or } 2 \text{ or } 3 \ldots). \qquad \text{II.83}$$

Keeping in mind that the summation must include fifty terms corresponding to natural frequencies greater than the highest driving frequency, the values of l_{max}, m_{max}, and n_{max} should be inflated to account for these extra terms.

After values of the natural frequency are computed for all mode shapes of (l,m,n), the list of natural frequencies must be ordered, keeping the corresponding indicies associated with the respective frequencies.

orthogonal series requires knowledge of the natural frequencies and their order.

The natural frequencies for a room of dimensions x_L, y_L, z_L are given by the equation

$$f_{l,m,n} = c/2 \ \{ (l/x_L)^2 + (m/y_L)^2 + (n/z_L)^2 \}^{1/2} , \qquad \text{II.79}$$

where c is the speed of sound in meters per second, which is

$$c = 331.6 \ \{ (273 + T)/273 \}^{1/2} , \qquad \text{II.80}$$

and T is the temperature in Centigrade

These frequencies are computed by chosing values for l, m, and n for a given room. As shown in Illustration 1, for (l,m,n) of (1,0,0), the frequency will correspond to a mode shape where there is one nodal plane which lies at the center of the x-axis in the y-z plane. A nodal plane or surface is the surface of a natural frequency response where acoustic pressure is low. Similarly, for (l,m,n) of (2,1,0), the frequency will correspond to a mode shape with three nodal planes: one at the center of the y-axis in the x-z plane, and two nodal planes on the x-axis, located at the first and third quarter points, in the y-z plane.

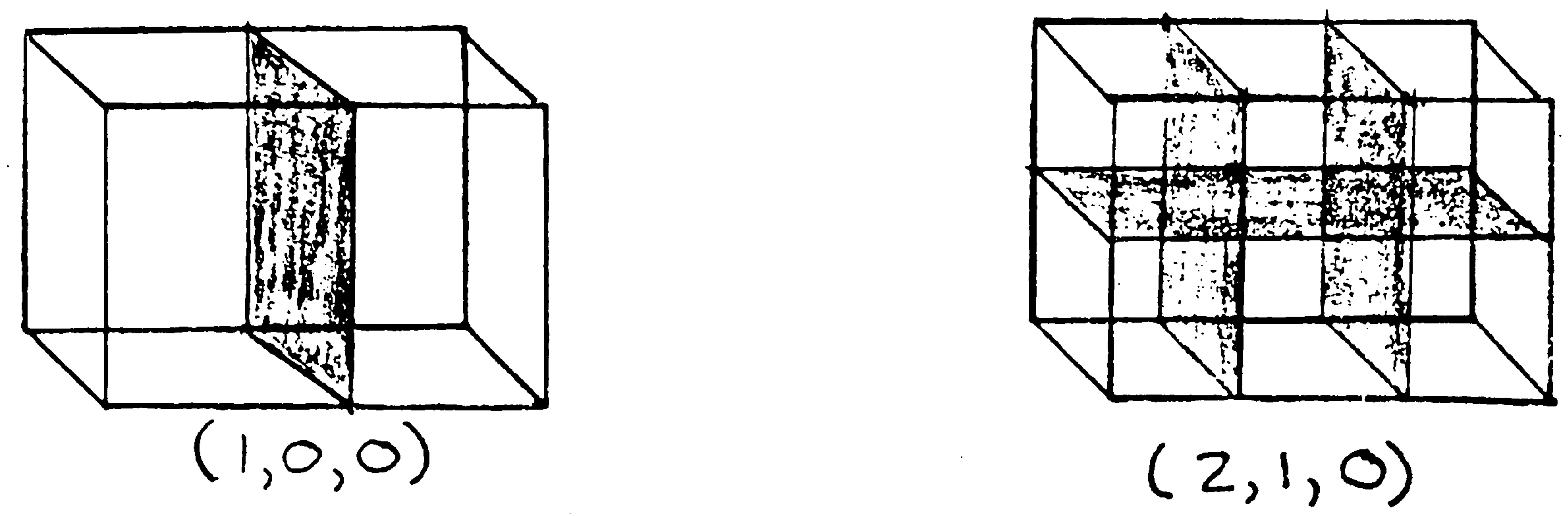


Illustration 1. Mode Shapes (1,0,0) and (2,1,0)

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After values of the natural frequency are computed for all mode shapes of (l,m,n), the list of natural frequencies must be ordered, keeping the corresponding indicies associated with the respective frequencies.

II. C. 2. Summing Terms Corresponding to Natural Frequencies Near the Driving Frequency

Figure 1 is a flow chart showing how the terms in the orthogonal series corresponding to the natural frequencies near the driving frequency are summed. To include the appropriate terms in the summation, the driving frequency is compared against the ordered list of the natural frequencies to determine which natural frequency is nearest to the driving frequency. Then terms corresponding to the fifty modes lower and fifty modes greater than this natural frequency are added into the summation, except if the nearest natural frequency is less than fifty, in which case the summation starts at the first mode and continues through the terms from the fifty greater modes.

For example, it is determined that the driving frequency is closest to the i^{th} natural frequency. The summation is begun by including the contribution from the $(i\text{-}50)^{th}$ mode with the (l,m,n) corresponding to that mode shape. Next the contribution is taken from the $(i\text{-}49)^{th}$ mode, using the indicies of (l,m,n) corresponding to that mode shape. Hence, what is really being done is just one summation over the natural frequencies of (i-50) to (i+50), instead of three summations over some ranges of l, m, and n.

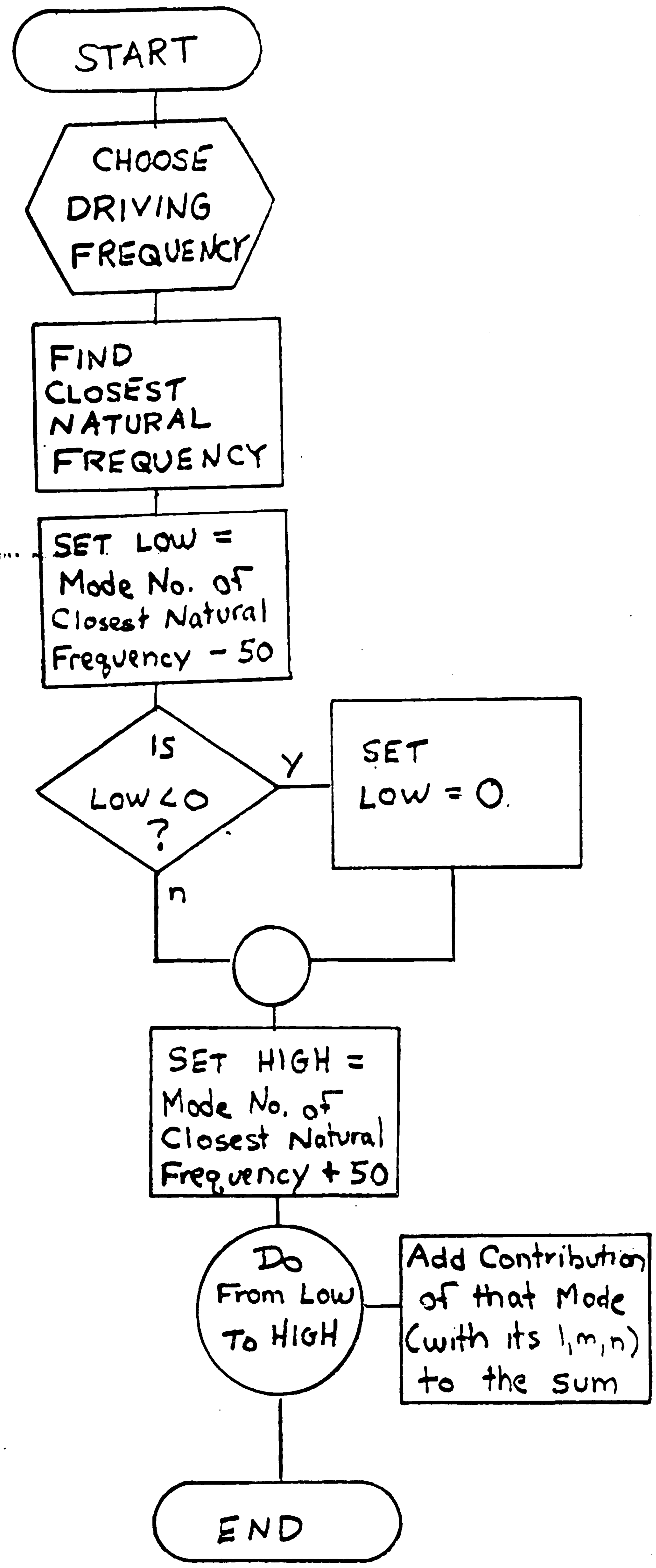


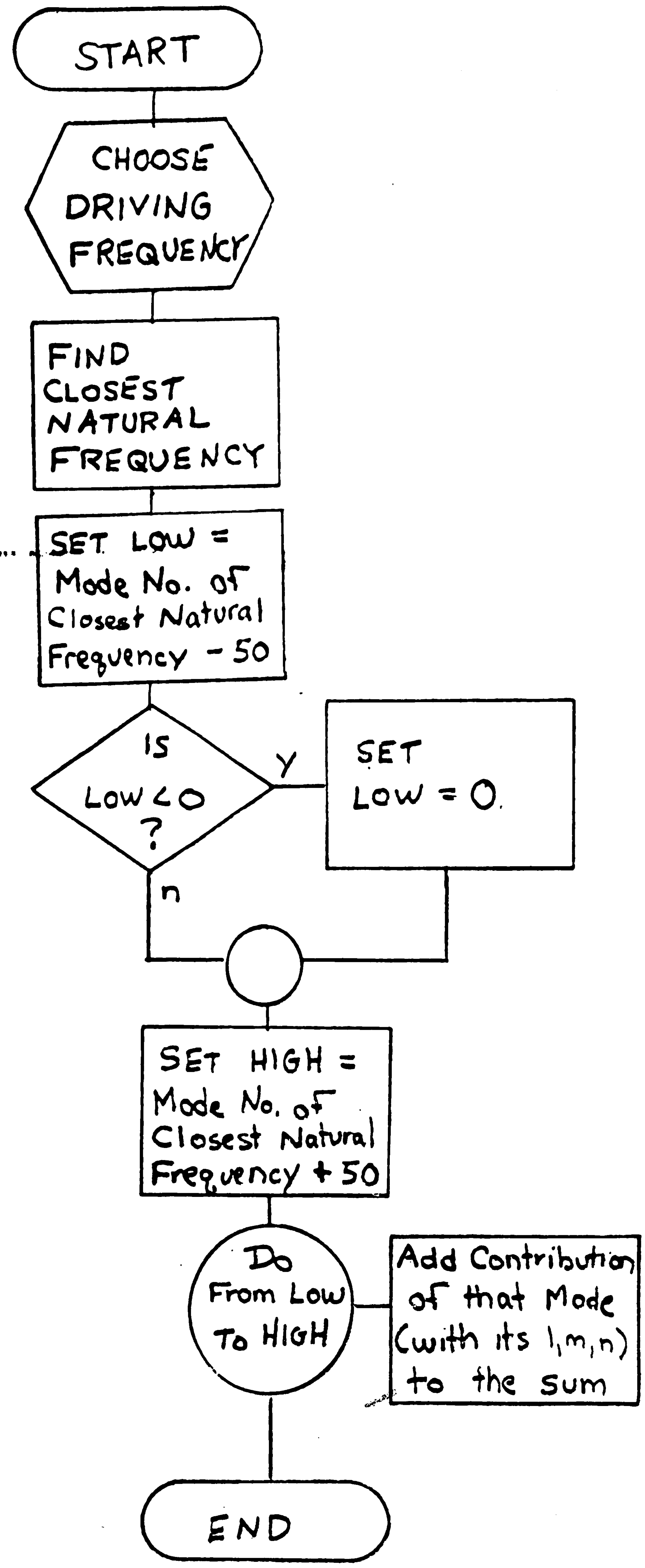
FIGURE 1. Flowchart for Summing Series Solution

II. C. 3. Computing Pressure in Decibels

After the summation is completed, it should be multiplied by the coefficients outside of the summation. This will yield a complex number for the pressure, which indicates the phase of this point in space with respect to the source. The pressure level in decibels is given by 20 times logarithm of the absolute value of the complex pressure value divided by a reference pressure:

$$\text{Pressure in dB} = 20.\ \log_{10}\left(\left| i\omega\rho Q_{\circ}\ \Sigma_{n-50}^{n+50} A_i \psi_i \right| / P_{ref} \right). \qquad \text{II.84}$$

To compute the amplitude versus frequency spectrum, the root mean square (rms) of the acoustic pressure values is required and the square root of two must be included in the denominator with the reference pressure.



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III. THE FINITE ELEMENT SOLUTION

III. A. DEVELOPMENT OF THE FINITE ELEMENT EQUATIONS

The differential equation for a point source located at $r=r_\circ$ is

$$\Delta p + k^2 p + i\omega\rho\, Q\delta(r-r_\circ) = 0 \,. \qquad \text{III.1}$$

(See ref [29] and ref [22] for internal and wall sources.) Using Galerkin's technique [27], multiply both sides by a weighting function $N_i(r)$ and integrate over the volume to obtain

$$\int_V \,[\Delta p + k^2 p + i\omega\rho\, Q\delta(r-r_\circ)]\; N_i(r)\; dV = 0 \,. \qquad \text{III.2}$$

In three dimensional cartesian coordinates, this is

$$\int_V [\partial^2 p/\partial x^2 + \partial^2 p/\partial y^2 + \partial^2 p/\partial z^2 + k^2 p$$

$$+ i\omega\rho\, Q\delta(x-x_\circ, y-y_\circ, z-z_\circ)]\; N_i(x,y,z)\; dV = 0 \,. \qquad \text{III.3}$$

Using Green's Theorem [32], the first term becomes

$$\int_V (\partial^2 p/\partial x^2)\; N_i(x,y,z)\; dV =$$

$$\int_S (\partial p/\partial x)\; N_i(x,y,z)\; \nu_x dS - \int_V (\partial p/\partial x)\; (\partial N_i/\partial x)\; dV \,, \qquad \text{III.4}$$

where ν_x is the x component of the unit normal vector $\bar{\nu}$ to the bounding surface S. Treating the second and third terms in a similar manner, Equation III.3 becomes

$$\int_V \{ (\partial p/\partial x)(\partial N_i/\partial x) + (\partial p/\partial y)(\partial N_i/\partial y)$$

$$+ (\partial p/\partial z)(\partial N_i/\partial z) - k^2 p N_i \}\ dV -$$

$$\int_S \{ (\partial p/\partial x)\nu_x + (\partial p/\partial y)\nu_y + (\partial p/\partial z)\nu_z \}\ N_i\ dS =$$

$$i\omega\rho Q \int_V \delta(x-x_\circ, y-y_\circ, z-z_\circ)\ N_i(x,y,z)\ dV\ . \qquad \text{III.5}$$

This can be easily condensed in a short hand notation to

$$\int_V \{ (\partial p/\partial r)(\partial N_i/\partial r) - k^2 p N_i \}\ dV - \int_S \{\partial p/\partial \nu\}\ N_i\ dS$$

$$= i\omega\rho Q \int_V \delta(r-r_\circ)\ N_i(r)\ dV\ . \qquad \text{III.6}$$

Along the bounding surface S, energy is absorbed, as expressed by

$$\partial p/\partial \nu + (i\omega\beta/c)\ p = 0\ , \qquad \text{III.7}$$

where β is the coefficient of absorption and $\partial p/\partial \nu$ is the gradient of p in the direction of $\bar{\nu}$. Incorporating this boundary condition yields

$$\int_V \{ (\partial p/\partial x)(\partial N_i/\partial x) + (\partial p/\partial y)(\partial N_i/\partial y)$$

$$+ (\partial p/\partial z)(\partial N_i/\partial z) - k^2 p N_i \}\ dV + (i\omega/c) \int_S \beta p\ N_i\ dS$$

$$= i\omega\rho Q \int_V \delta(x-x_\circ, y-y_\circ, z-z_\circ)\ N_i(x,y,z)\ dV\ . \qquad \text{III.8}$$

Integrating the right hand side yields

$$\int_V \{ (\partial p/\partial x)(\partial N_i/\partial x) + (\partial p/\partial y)(\partial N_i/\partial y)$$

$$+ (\partial p/\partial z)(\partial N_i/\partial z) - k^2 p N_i \}\ dV + (i\omega/c) \int_S \beta p\ N_i\ dS$$

$$= i\omega\rho Q_\circ N_i(x_\circ, y_\circ, z_\circ)\ . \qquad \text{III.9}$$

The finite element approximation assumes that the pressure can be expressed as a product of the finite element interpolating functions and the nodal values, i.e.

$$p = [N]\ \{P\}\ , \tag{III.10}$$

where $[N]$ is a row matrix whose entries are the interpolating functions $N_i(x,y,z)$ (coincident with weighting functions) and $\{P\}$ is the vector of nodal point values. This can be inserted into Equation III.9 as

$$\Big(\int_V \big((\partial N_i/\partial x)[\partial N/\partial x] + (\partial N_i/\partial y)[\partial N/\partial y] + (\partial N_i/\partial z)[\partial N/\partial z] - k^2 N_i [N] \big)\, dV + (i\omega/c) \int_S \beta N_i [N]\, dS \Big)\{P\} = i\omega\rho Q_\circ N_i(x_\circ, y_\circ, z_\circ)\ , \tag{III.11}$$

This may be written in abbreviated form as

$$[K]\{P\} = \{R\}\ , \tag{III.12}$$

where
$$K_{ij} = \int_V \big((\partial N_i/\partial x)(\partial N_j/\partial x) + (\partial N_i/\partial y)(\partial N_j/\partial y) + (\partial N_i/\partial z)(\partial N_j/\partial z) - k^2 N_i N_j \big) dV + (i\omega/c)\int_S \beta\, N_i N_j\, dS\ , \tag{III.13}$$

and
$$R_i = i\omega\rho Q_\circ N_i(x_\circ, y_\circ, z_\circ)\ . \tag{III.14}$$

$[K]$ may be noted to be a complex valued symmetric matrix.

Unfortunately, ω, the driving source frequency introduced by the inclusion of the wall absorption term, appears in the formulation of the matrix $[K]$. This means that the matrix must be reformed and decomposed for every new value of driving frequency. Similarly, β appears in the matrix, so for any change in the wall absorption coefficient, the matrix must also be reformed and decomposed. The only parameter changes for which one does not have to reform and decompose the $[K]$ matrix are changes in the density or source strength, which produces a linearly proportional effect, or changes in the location or number of sources.

III. B. NATURAL FREQUENCIES BY THE FINITE ELEMENT METHOD

The finite element technique may also be use to determine the natural frequencies and mode shapes of the enclosure. The equation for the room with soft walls without a source is

$$[K] - k^2[M] + k\ [B] = 0\ , \qquad \text{III.15}$$

where $$K_{ij} = \int_V \{\ (\partial N_i/\partial x)(\partial N_j/\partial x) + (\partial N_i/\partial y)(\partial N_j/\partial y) +$$

$$(\partial N_i/\partial x)(\partial N_j/\partial x)\ \}\ dV\ , \qquad \text{III.16}$$

$$M_{ij} = \int_V N_i N_j\ dV\ , \qquad \text{III.17}$$

and $$B_{ij} = i \int_S N_i N_j\ dS\ . \qquad \text{III.18}$$

The solutions for k must be found using a numerical method of solving for these Eigenvalued roots. Most likely, the technique would involve guessing and iterating a value of k until its value was known within error limits. Then, after the k_i were known, the natural frequencies of the room would appear as

$$f_{nat} = (k_i c)/(2\pi)\ , \qquad \text{III.19}$$

since $k = \omega/c$, III.20

and $f_{nat} = \omega/2\pi$. III.21

The effect of the soft walls on the Eigenvalues is negligible for most situations. Dropping the absorption matrix [B] from the Equation III.15 yields a much simpler problem to solve. For hard walls, the Generalized Eigenproblem can be written as

$$[K] - k^2[M] = 0\ . \qquad \text{III.22}$$

III. C. COMPUTER IMPLEMENTATION OF THE FINITE ELEMENT SOLUTION [29]

The finite element equation for this model was stated as

$$[K]\{P\} = \{R\} , \qquad \text{III.23}$$

where $$K_{ij} = \int_V (\partial N_i/\partial r)(\partial N_j/\partial r) - k^2 N_i N_j \partial V + (i\omega/c)\int_S \beta\, N_i N_j\, \partial S \qquad \text{III.24}$$

and $$R_i = i\omega\rho Q_\circ N_i(r_\circ) . \qquad \text{III.25}$$

This is a complex symmetric matrix problem.

III. C. 1. RST mapping and Gaussian Integration

To solve this problem by finite element analysis, elements appropriate to the integrand must first be chosen. The order of continuity required of the interpolating functions is one less than the order of the highest derivative appearing in the integrand. Note that the integrand contains only first order derivatives

$$(\partial N_i/\partial r)(\partial N_j/\partial r) ,$$

which implies that only c^o continuity is required of the interpolating functions. Thus, linear elements may be used. Three dimensional eight node linear isoparametric brick elements were chosen for this development because they are the simplest to construct, can be used to model shapes which vary vastly from one to the next, keep down the number of nodes (which plagues three dimensional problems), and are therefore the most economical to use.

The basis of isoparametric formulation is to use the same function for interpolating functions as those which define the coordinate transformation. The six-sided, eight node brick may be obtained by a point mapping from a cube, where the brick is known as a global element in the global coordinate system. The cube is the local element in the local coordinate system. Straight lines from the local coordinate system map to straight lines in the global coordinate system.

This mapping and interpolation function is a row vector:

$$[\ N(r,s,t)\ ] = \qquad \text{III.26}$$

$$1/8\ \ [(1+r)(1+s)(1+t)\ \ (1-r)(1+s)(1+t)\ \ (1-r)(1-s)(1+t)\ \ (1+r)(1-s)(1+t)$$
$$(1+r)(1+s)(1-t)\ \ (1-r)(1+s)(1-t)\ \ (1-r)(1-s)(1-t)\ \ (1+r)(1-s)(1-t)]$$

Note that $p(x,y,z) = [\ N(r,s,t)\ ]\{\bar{p}\}$, III.27

but $\partial N/\partial x$, $\partial N/\partial y$, and $\partial N/\partial z$ is needed. Using the chain rule

$$\begin{Bmatrix} \partial/\partial r \\ \partial/\partial s \\ \partial/\partial t \end{Bmatrix} = \begin{bmatrix} \partial x/\partial r & \partial y/\partial r & \partial z/\partial r \\ \partial x/\partial s & \partial y/\partial s & \partial z/\partial s \\ \partial x/\partial t & \partial y/\partial t & \partial z/\partial t \end{bmatrix} \begin{Bmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{Bmatrix}, \qquad \text{III.28}$$

where $\partial x/\partial r = 1/8\ \ [(1+s)(1+t)x_1\ -(1+s)(1+t)x_2\ -(1-s)(1+t)x_3\ \ (1-s)(1+t)x_4$

$$(1+s)(1-t)x_5\ -(1+s)(1-t)x_6\ -(1-s)(1-t)x_7\ \ (1-s)(1-t)x_8]. \qquad \text{III.29}$$

The matrix in Equation III.28 is called the Jacobian Matrix. Solving for the needed values of $\partial N/\partial x$, $\partial N/\partial y$, and $\partial N/\partial z$ yields

$$\begin{Bmatrix} \partial N/\partial x \\ \partial N/\partial y \\ \partial N/\partial z \end{Bmatrix} = [\ J\ ]^{-1} \begin{Bmatrix} \partial N/\partial r \\ \partial N/\partial s \\ \partial N/\partial t \end{Bmatrix}. \qquad \text{III.30}$$

III. C. 2. Solving the Volume Integral

At this point, the integrals of Equation III.24 can now be solved numerically using Gaussian Integration and the above development. From a theorem of advanced calculus [27], the integration of a function over an element is

$$\iiint_e f()\ \partial x \partial y \partial z = \int_{1}^{1} \int_{1}^{1} \int_{1}^{1} f()\ |J|\ \partial r \partial s \partial t\ , \qquad \text{III.31}$$

where $|J|$ is the determinant of the Jacobian Matrix. In this case, the volume integral is

$$\int_V (\partial N_i/\partial r)(\partial N_j/\partial r) - k^2 N_i N_j \partial V$$

and the function is

$$f() = (\partial N_i/\partial x)(\partial N_j/\partial x) + (\partial N_i/\partial y)(\partial N_j/\partial y) + (\partial N_i/\partial z)(\partial N_j/\partial z)$$
$$- k^2 N_i N_j\ . \qquad \text{III.32}$$

The numerical technique of Gaussian Second Order Integration [27] can be used easily to integrate this to yield the following:

$$\int_{1}^{1} \int_{1}^{1} \int_{1}^{1} f() \partial x \partial y \partial z = \Sigma_{i=1}^{2}\ \Sigma_{j=1}^{2}\ \Sigma_{k=1}^{2}\ f(r_i, s_j, t_k)|J|\ . \qquad \text{III.33}$$

Here, the Gauss weights equal unity and the Gauss points are

$$r_1 = s_1 = t_1 = -1/\sqrt{3}\ , \qquad \text{III.34}$$

$$r_2 = s_2 = t_2 = +1/\sqrt{3} \qquad \text{III.35}$$

and it was previously ascertained that

$$N_i = N_i(r,s,t) = N(r_i, s_i, t_i)\ . \qquad \text{III.36}$$

III. C. 3. Solving the Surface Integral

The integration of the surface integral of Equation III.24

$$(i\omega/c)\int_S \beta\, N_i N_j\, \partial S \ ,$$

proceeds in a manner similar to the integration of the volume integral, but in two dimensions. There is a slight complication in that the nodes are expressed in three dimensional space instead of two dimensional space. Therefore, three dimensional space must first be mapped to an intermediate two dimensional space before mapping back and forth between the local and global coordinate systems.

Furthermore, an ordering rule for numbering both the sides and the nodes so that the model is consistent with the program must be established. The numbering of the sides is in terms of the node numbers and the ordering rule established for the local element applies to all global elements. For instance, side 1 is bounded by nodes 1, 2, 3, 4, going counter-clockwise, and side 4 is bounded by nodes 6, 5, 8, 7, counter-clockwise.

```
  6 -- 5
 /|   /|      s                     6 4
2 + -- 1 |    |                     |/
| |   | |  in |__ r  with sides 2 --+-- 5
| 7 -- + 8   /                     /|
|/    |/    t                     1 3
3 -- 4
```

Illust. 2. Ordering Scheme for Nodes and Sides of the Local Element

The development is the same as that of the volume integral. The function of the integrand is

$$f() = \beta\, N_i N_j \ ,$$

and the surface integral becomes

$$\sum_{ir=1}^{2} \sum_{is=1}^{2} \{\beta\, N_i(r_{ir},s_{is})\, N_j(r_{ir},s_{is})\}\ |J| \ ,$$

where the Gauss points and weights are the same as in the preceeding section.

An unusual characteristic to these elements is that absorption is assigned to a side of an element, rather than to the nodes. This relates a physical property to the sides of an element, rather than to the nodes. It is very important to keep track of where the sides of the elements are. In this mesh, one array is needed to store the eight global nodes of every element, and another array to store the absorption on the six sides of every element.

III. C. 4. Solving the Matrix

Lower-Upper (L-U) Decomposition with complex numbers for symmetric matrix may be used to solve this. The Crout Reduction scheme [23] is recommended because the transformed lower and upper matrices can be stored in the same storage location as the original matrix, and the transformed matrix will be banded if the original matrix was banded. Unfortunately, as mentioned earlier, the matrix must be reformed and decomposed for every new value of driving frequency or wall absorption. The only change for which one does not have to reform and decompose the matrix is for a change in the density or source strength, which will have a linearly proportional effect, or a change in the location or number of sources.

III. D. COMPUTING NATURAL FREQUENCIES FROM THE FINITE ELEMENT EQUATIONS

The Generalized Eigenproblem for finding the natural frequencies was stated as

$$[K] - k^2[M] = 0 \ . \qquad \text{III.37}$$

The matrix form for computing the natural frequencies after a development is similar to that above for the volume integral except

$$K_{ij} = \Sigma_{ir=1}^{2} \Sigma_{is=1}^{2} \Sigma_{it=1}^{2} (\partial N_i/\partial r)(\partial N_j/\partial r) \ |J| \ , \qquad \text{III.38}$$

and $$M_{ij} = \Sigma_{ir=1}^{2} \Sigma_{is=1}^{2} \Sigma_{it=1}^{2} N_i \ N_j \ |J| \ . \qquad \text{III.39}$$

Both of these matrices can be produced in the same subroutine from the same variables.

This calculation yields two real symmetric matrices, which give real eigenvalues from which the natural frequencies can be computed. As we mentioned above, the natural frequencies may be computed from the values obtained for k:

$$f_n = k_n c/2\pi \ . \qquad \text{III.40}$$

III. E. COMPUTER PROGRAMS

III. E. 1. Program to Compare Solutions (ACOMPAR.LU)

This algorithm (see Figure 2) was used to generate the solution to a rectangular enclosure with wall damping and a point source by two different methods, finite element analysis and the orthogonal series, so that both solutions could be compared, using the same physical constants.

The objective is to test the finite element model against the orthogonal series solution over a range of driving frequencies and absorption coefficients. The procedure is begun by loading the mesh information, computed in a mesh writing program, into this routine. This indicates where the nodes are, how they are connected, and which element sides touch which exterior absorption surfaces.

Constants for the computer run are assigned, including the physical characteristics which are not of interest in varying (e.g., air temperature and density, reference pressure, source strength, along with some numerical constants of pi and the square root of two). Next, the program computes the bandwidth of the matrix, based upon the connection of the nodes by the elements. And for the orthogonal series solution, the program computes the dimensions, volume, natural frequencies, and mode shapes of the room.

Now the room is examined over the ranges which are of interest in varying. The program starts with lower frequencies, increasing them by sampling at the natural frequencies and half way between the natural frequencies. It computes ω and k based upon the value of these chosen

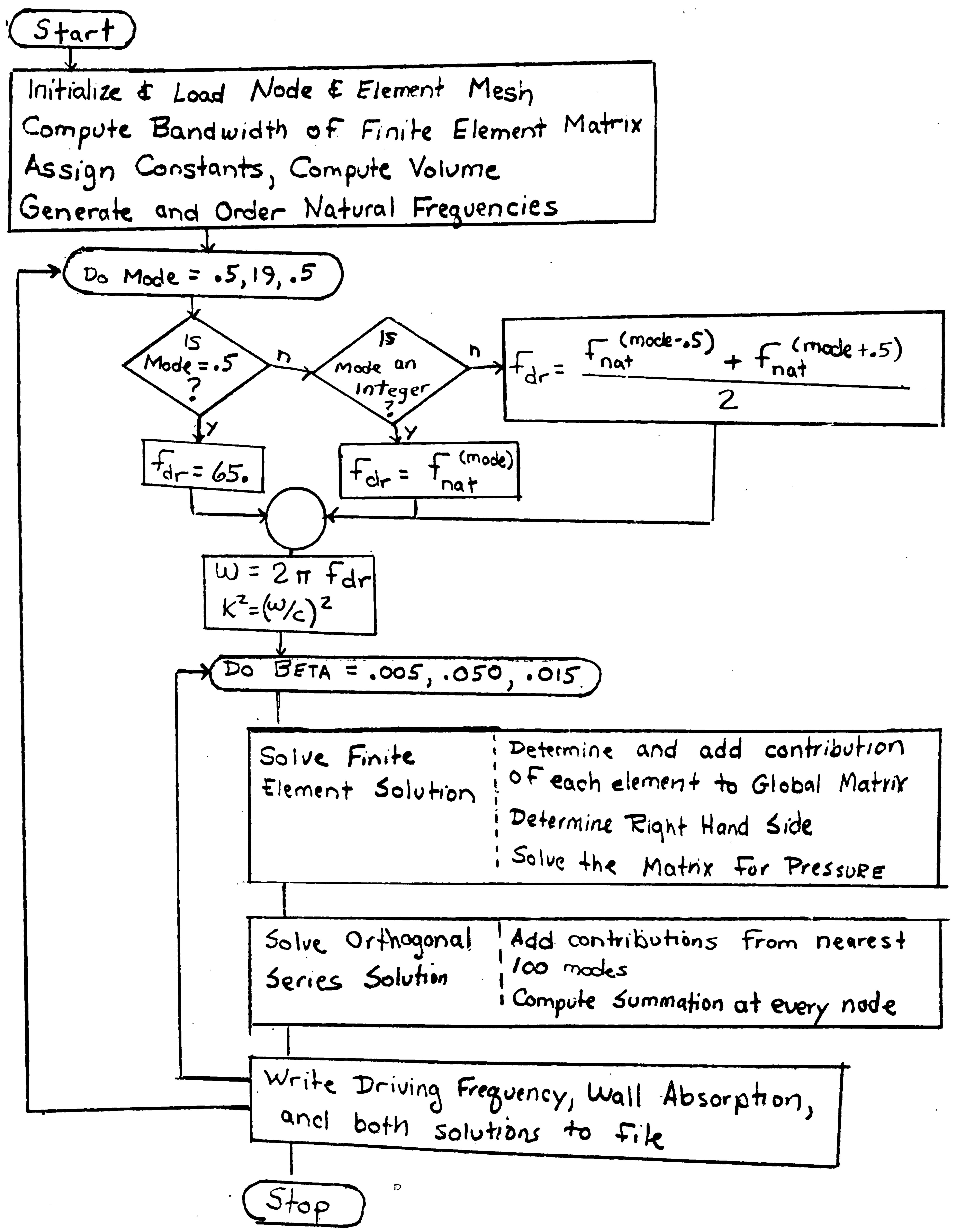


Figure 2. Flowchart for ACOMPAR.LU

driving frequencies. Inside this loop, it starts an interior loop in which increases the absorption from relatively hard walls to much softer walls, going from an absorption value of 0.005 to 0.050 in three increments.

Next, the program computes the finite element solution. It computes the volume and surface integrals for each element and add these values to the global matrix for the current values of driving frequency and absorption. Then it decomposes the matrix, and solves it with the right hand side for the source at one of the nodes.

The program then calculates the pressure with the closed form solution, by computing the summation for pressure at one nodal location with the source at the driving node. Computing the summation involves summing the contribution of neighboring mode shapes of natural frequencies near the driving frequency. The pressure is computed this way at every node.

Both solutions are written to a file and a new value of absorption and/or driving frequency is selected. After all absorptions are computed at one driving frequency, a new driving frequency is selected and it starts again with the lower value of absorption. The time required to analyze just one driving frequency with one absorption varies from about one minute for a mesh with 125 nodes to about eighteen minutes for a mesh of 1728 nodes on a VAX 11/780 minicomputer.

III. E. 2. A Finite Element System for Acoustics (ACOSÝS.LU)

An algorithm has been developed to manipulate a node and element mesh of any complex shaped enclosure, to process that mesh for acoustic pressure or natural frequencies, and to display the results on a color computer graphics terminal. The system is outlined in Figure 3.

This system starts by printing a title block to the screen to acknowledge to the user that he has entered the Acoustic System. The program will immediately initialize variables to prevent the computer from trying to display uninitialized variables.

This system is menu driven. The first menu a user sees is not the MAIN menu, but the LOAD MESH menu, which allows a user to specify a file with an acoustic finite element mesh. A mesh need not be loaded at this time as the program has initialized all the variables, but a mesh written previously to entering the system must be loaded in at some point before the computer can display old results or compute new results. The system currently has no mesh generator, so this task must be done outside the Acoustic System. Mesh generation is discussed in more detail in Section IV. D. and must conform to the format specified in the HELP menu of LOAD MESH.

After a mesh is loaded, the user can select to get to the MAIN menu, or a see a help, which shows the format required for the mesh file. Entering the MAIN menu, the user can select a number of options, including loading another mesh file, getting results out of the mesh file, changing variables, processing the model for either acoustic pressure at the nodes or for the natural frequencies, writing results to a file, or terminating the session.

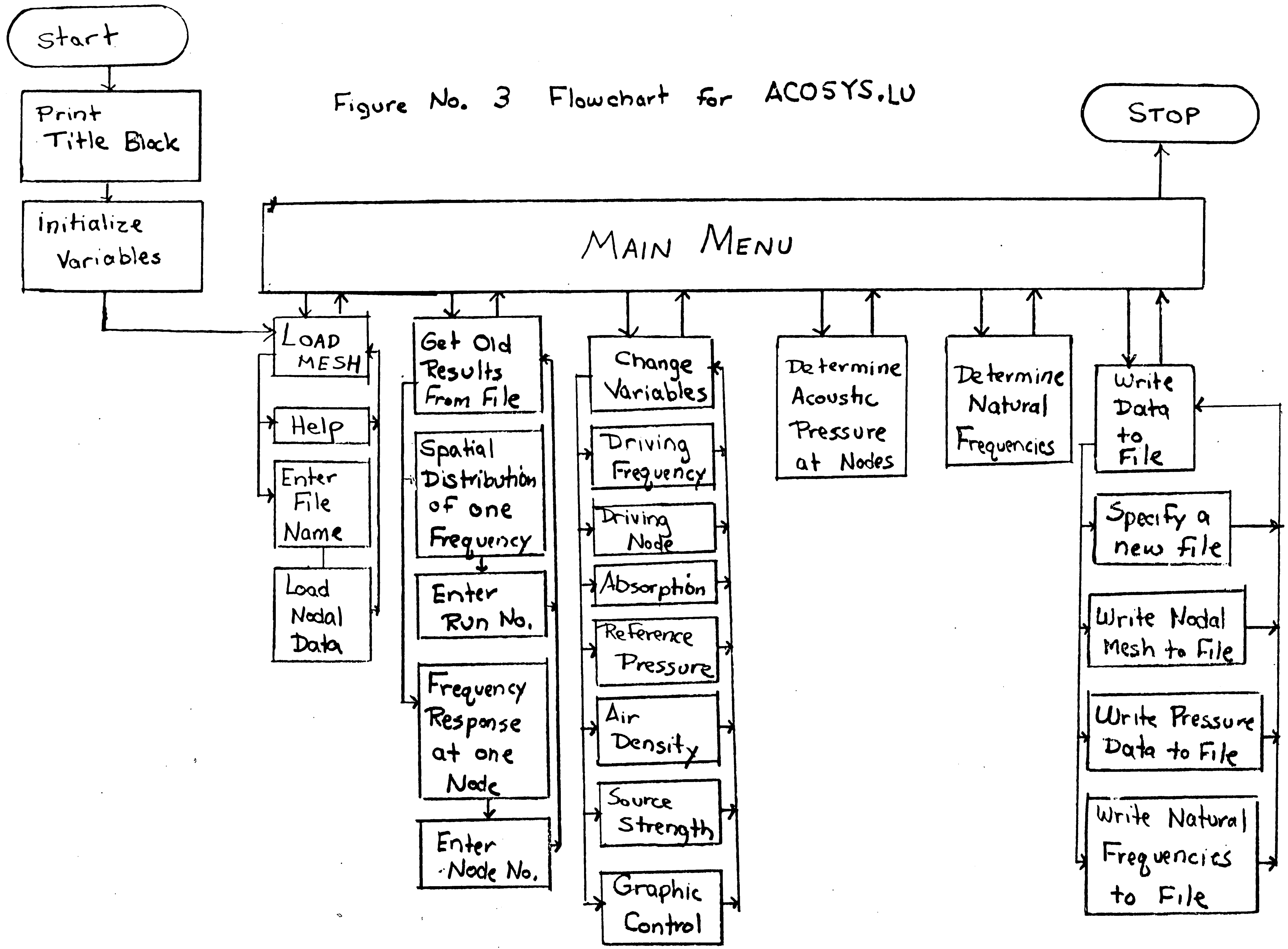


Figure No. 3 Flowchart for ACOSYS.LU

The program is capable of getting results from a mesh file if the results are stored after the mesh information. This precaution ensures that the results do not get separated from the mesh from which they were created. Thus, a user can make several runs in advance of entering the acoustic system and call up the spatial distribution at one of those driving frequencies or absorption coefficients, or if all the runs have the same absorption coefficient, to display the frequency response at one node. He could also recall the results from a previous Acoustic System session.

The system allows the user to change variables in the CHANGE VARIABLES menu and to make another run. Thus, a designer can investigate the spatial response at another frequency or the effect a different absorption may have on the spatial or frequency response. The driving node or frequency may be changed. (Currently there is no provision in the program for handling multiple sources at the same frequency. And the finite element solution presented in this paper does not account for multiple sources at different frequencies.) Source strength, density, temperature, and reference pressure may also be changed. The program also has a graphics controls key for selection of the desired graphics format.

The program has the capability to process the model with the conditions assigned in CHANGE VARIABLES and to then graphically display the results. Alternatively, the program could compute the natural frequencies.

In the WRITE menu, the system allows a user to specify a new file name, so that information could be written to several files. Nodal data and acoustic pressure could be written in a format that can be recalled by the system at a later time, and displayed graphically in the GET OLD RESULTS menu. Natural frequencies can be written to a file so that the designer can have a hard copy of these important frequencies.

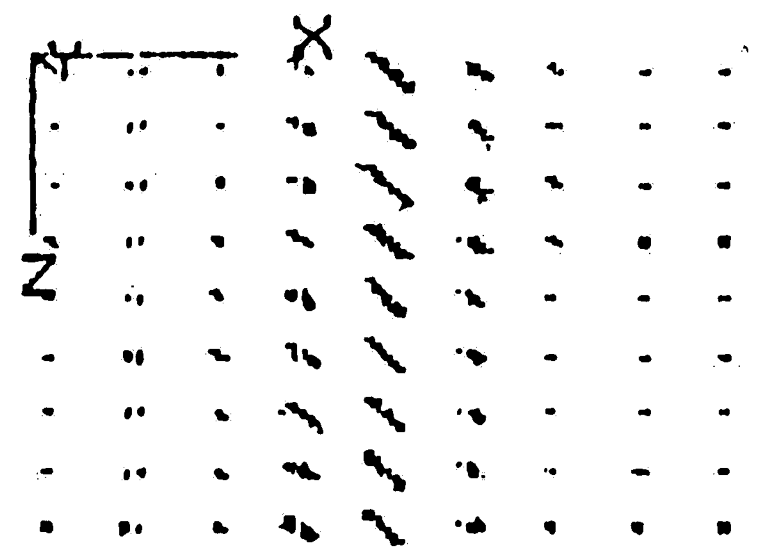
Upon termination of the Acoustic System from the MAIN menu, the system clears all graphics left up on the screen and terminates the system, returning the user to the operating system.

For the spatial response graphics, the system displays three dimensional data and colors the nodal points in space according to the acoustic pressure, starting with the low pressure, and filling up slowly to the higher acoustic pressure. The process may be temporarily suspended by pressing the NO SCROLL key to examine the reverberant space as it is filling in increasingly higher presure levels. Pressing the NO SCROLL key a second time allows the computer to continue to fill in with the next pressure levels. Figure 4 shows a spatial distribution which was interupted after the level of 101 decibels was displayed.

The frequency response graphics produces a graph of the acoustic pressure in decibels versus the frequency in Hertz. The pressure level at a selected node will be displayed as a point on the graph at the associated frequency. Figure 5 is a frequency response for a trapezoidal room with 125 nodes.

FREQUENCY RESPONSE
FINITE ELEMENT SOLUTION

DRIVING FREQUENCY = 136 HZ
SPEAKER POSITION
X= 0 Y= 0 Z= 0
WALL NO. = 1 ABSORPTION = 5.E-3



69 80 90 101 DECIBELS

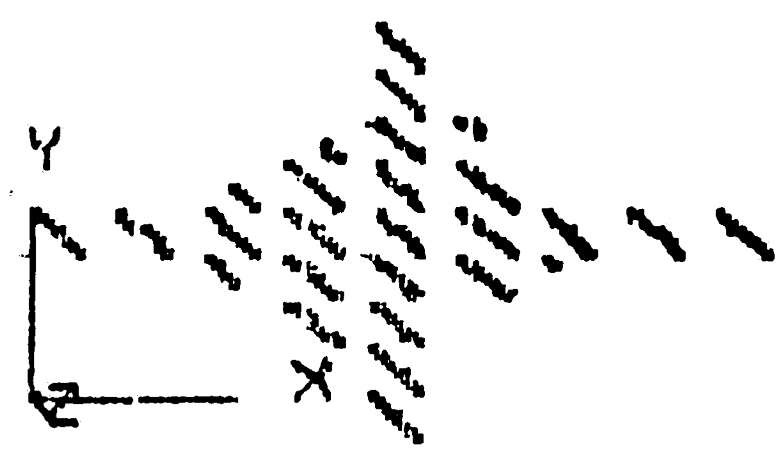


Figure 4. Spatial Distribution Display

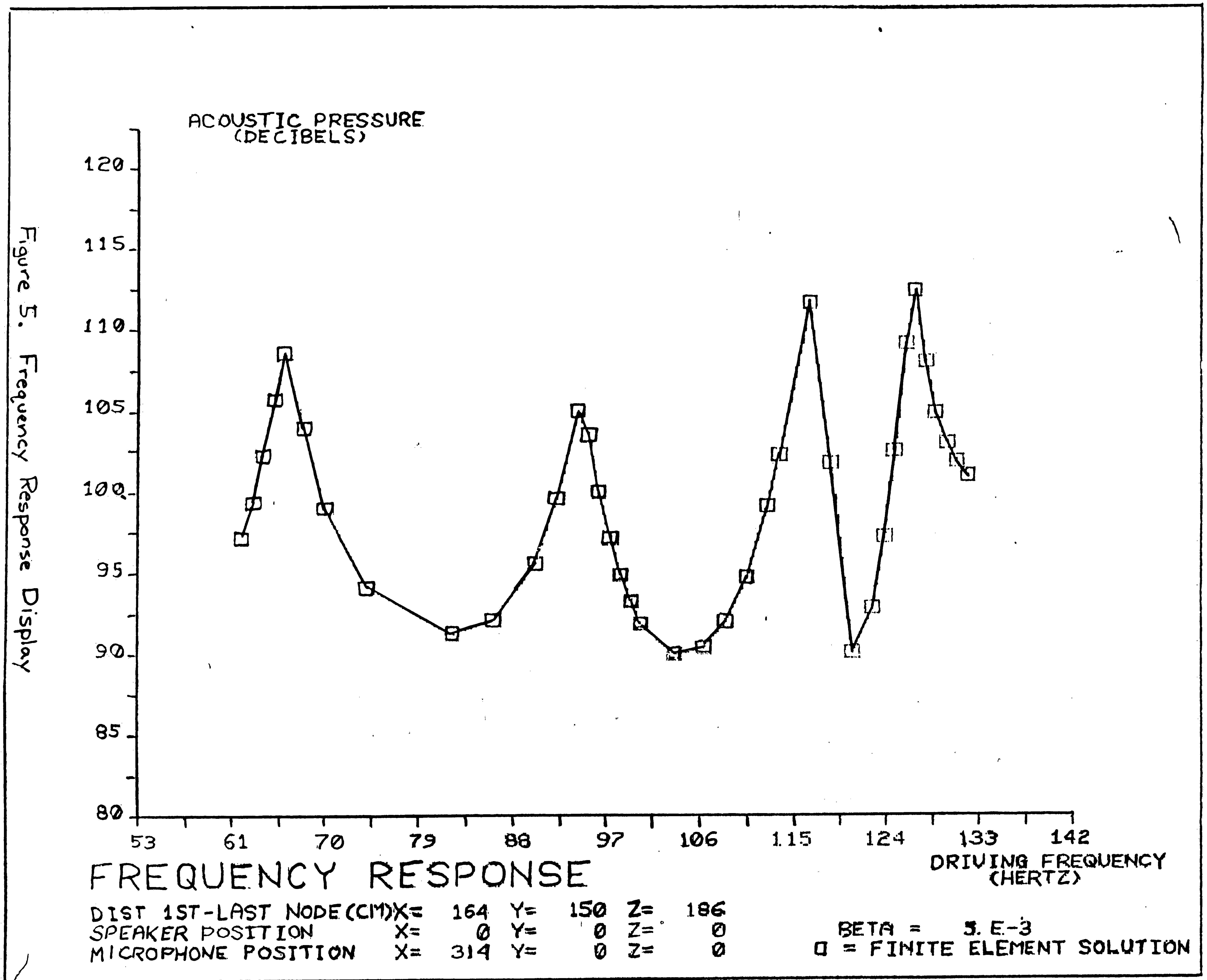


Figure 5. Frequency Response Display

IV. RESULTS

IV. A. COMPARISON OF THE TWO METHODS OF SOLUTION

IV. A. 1. General Agreement

The two solutions yield remarkably similar results at lower frequencies. However, at higher frequencies, the finite element solution peaks at slightly higher frequencies than the orthogonal series solution. This difference is due to the spacial sampling of the finite element method, and this shift increases with frequency. For a given volume, the more finite elements used to model that space, the closer the finite element model would approximate continuous space, and the less pronounced this shift would be.

This can be seen immediately in the results of the natural frequencies from the finite element equations. The solution anticipates the natural frequencies at a higher frequency than the continuous solution. Thus when the finite element space is driven at arbitrarily chosen frequencies, it peaks at frequencies corresponding to those inflated natural frequencies.

IV. A. 2. Specific Results from the Rectangular Room Example

The following table shows the location of the natural frequencies of continuous space and also for finite element space with different number of elements.

Table 1. Natural Frequencies (Hz) of Continuous and Finite Element Space

			Number of Nodes						
Mode	Shape	Cont	64	80	100	125	343	729	1728
1	1,0,0	72	76	74	74	72	72	72	72
2	0,0,1	91	97	97	95	-	91	91	91
3	0,1,0	115	120	120	120	-	-	115	-
4	1,0,1	116	123	122	121	-	116	116	116
5	1,1,0	136	142	141	141	140	136	136	136
6	2,0,0	145	155	155	153	-	151	-	147
7	0,1,1	147	169	159	159	-	-	147	-
8	1,1,1	164	172	171	170	167	167	164	164
9	2,0,1	171	195	187	187	x	177	-	-
10	0,0,2	183	207	200	200	x	-	-	-
11	2,1,0	185	217	217	204	x	191	191	-
12	1,0,2	196	229	222	218	x	-	201	196
13	2,1,1	206	230	229	221	x	211	211	-
14	0,1,2	216	239	248	237	x	x	-	222
15	3,0,0	217	248	259	248	x	x	222	222
16	1,1,2	228	258	259	259	x	x	-	-
17	0,2,0	230	259	269	259	x	x	234	233
18	2,0,2	232	268	269	269	x	x	-	235
19	3,0,1	235	269	277	276	x	x	-	-

Table 1 shows the natural frequencies for a rectangular room of length 2.396 m., height 1.505 m., and depth 1.865 m. It shows the mode, mode shape, and the natural frequencies as computed by the closed form orthogonal series solution, assumed to be exact. The values of the natural frequencies are estimated using the finite element method on this space using different numbers of nodes. For meshes of up to 100 nodes, the natural frequencies could be computed on a CDC Cyber 850 Mainframe. For meshes of more than 100 nodes, the natural frequencies had to be determined by examining frequency response curves and spatial distributions. Due to poor sampling, the location of all natural frequencies could not be placed. Dashes (-) indicate that there is a natural frequency, although its exact location could not be determined. X's indicate that the mode appears to be beyond the model's limits of validity. However, for less than 100 nodes, these frequencies were listed anyway.

The table shows that the location of the natural frequencies vary with the number of nodes in a model and with the mode number being examined. The more elements used to model a space of a given size, the closer the finite element model resembles continuous space, and the better that model is at predicting the location of the natural frequency compared with continuous space. The error increases in the positive direction with the mode number.

IV. A. 3. Spatial Distribution

Agreement of the spatial distribution at each of the points between the two solutions at a given frequency is excellent. Even with a very coarse mesh, both solutions behave similarly.

Figure 6 shows the acoustic pressure as it varies in the x-direction for the first mode shape, that is, the (1,0,0) mode appearing at 72 Hertz. In this mode shape, pressure is high at the y-z plane walls at x = 0, and x = 2.396. It forms a nodal plane of low pressure at $x = 1/2\ X_L = 1.198$ m. A microphone, moving along the x-axis through the centers of the y-z plane walls would record the following results, according to the respective theoretical model of different nodal density.

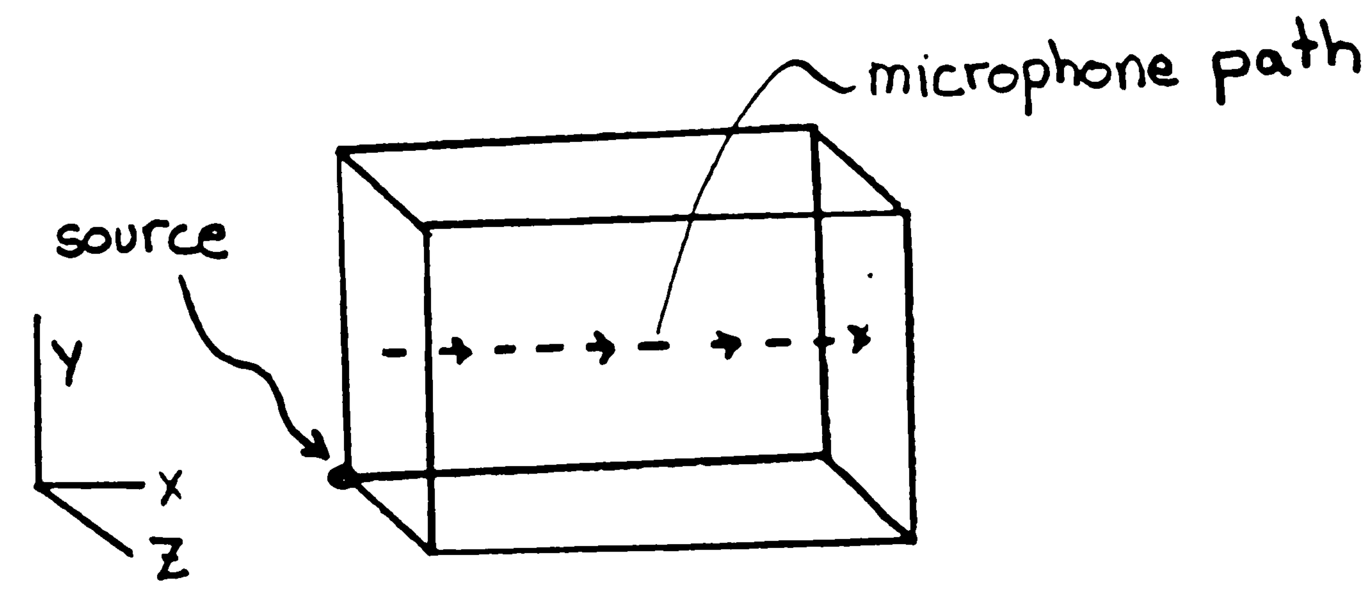


Illustration 3. Microphone Path Across Enclosure

Agreement is improved by increasing the number of nodes in any given model. With a coarse mesh of 125 nodes (5 along each axis), the finite element solution will produce the proper mode shapes through the first five natural frequencies with some accuracy. However, the maximum pressure at the walls by the finite element solution for the first mode was predicted to be 101.6 dB, 10 dB lower than the 111.5 dB predicted by the orthogonal series solution. By increasing the mesh to 1728 nodes (12 along each axis), the pressure computed by the finite element solution was 110.9 dB, only 0.6 dB low.

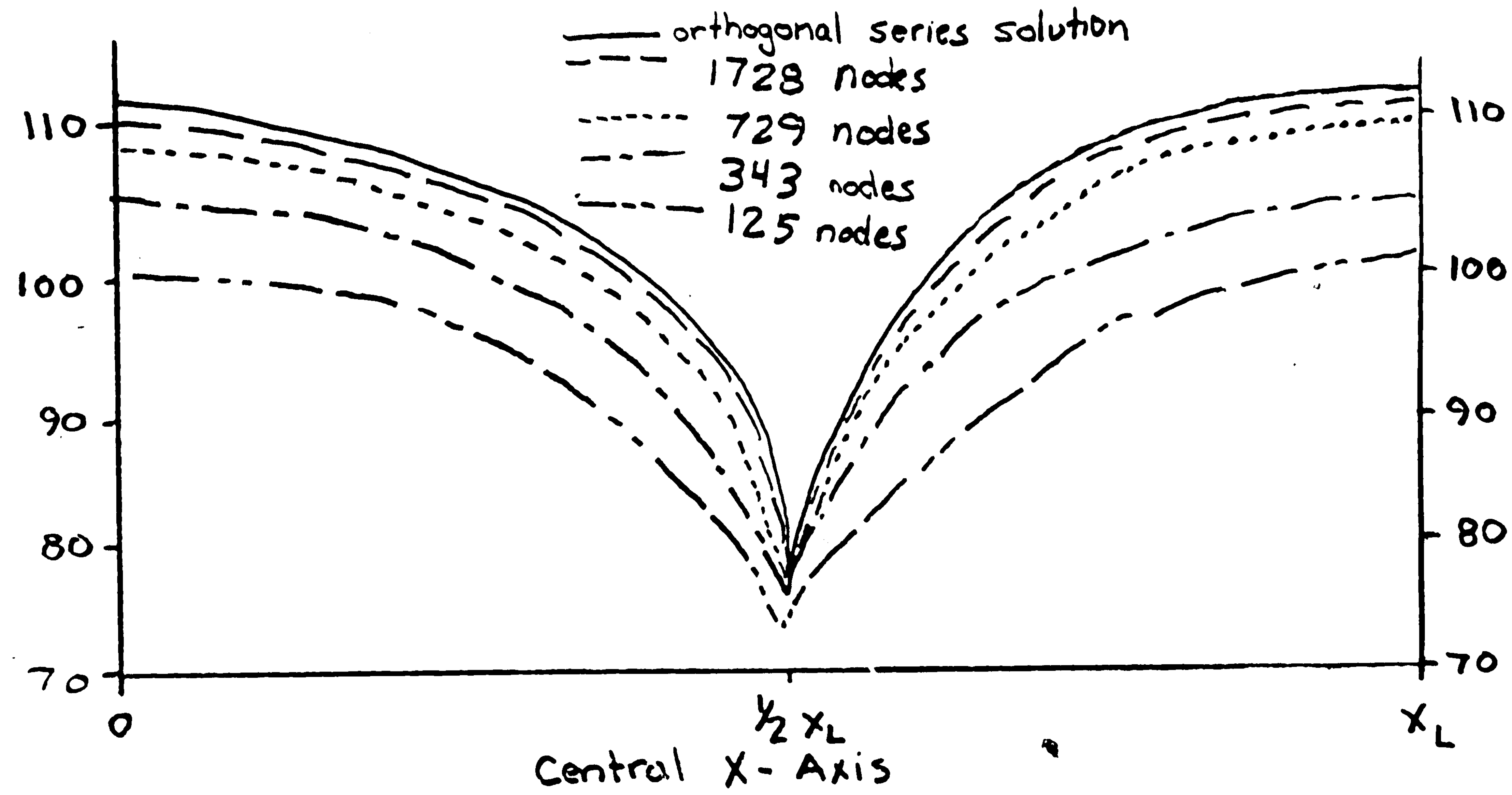


Figure 6. Acoustic Pressure Along Central X-Axis at 72. Hz (First Mode)

The improvement with increasing the number of nodes can be seen with its effects on the accuracy of the solution on the first five natural frequencies as Figure # below shows. Percent error is defined to be the average over all points of the difference of the acoustic pressure in decibels computed by the closed form solution minus the pressure computed by the finite element solution, divided by the closed form solution pressure, times 100. The general trend appears to that the error is small to begin with at these natural frequencies, and appears to diminish with an increased number of nodes along an axis.

$$\% \text{ Error} = (P_{cont} - P_{fem})/P_{cont} \times 100\% \qquad \text{IV.1}$$

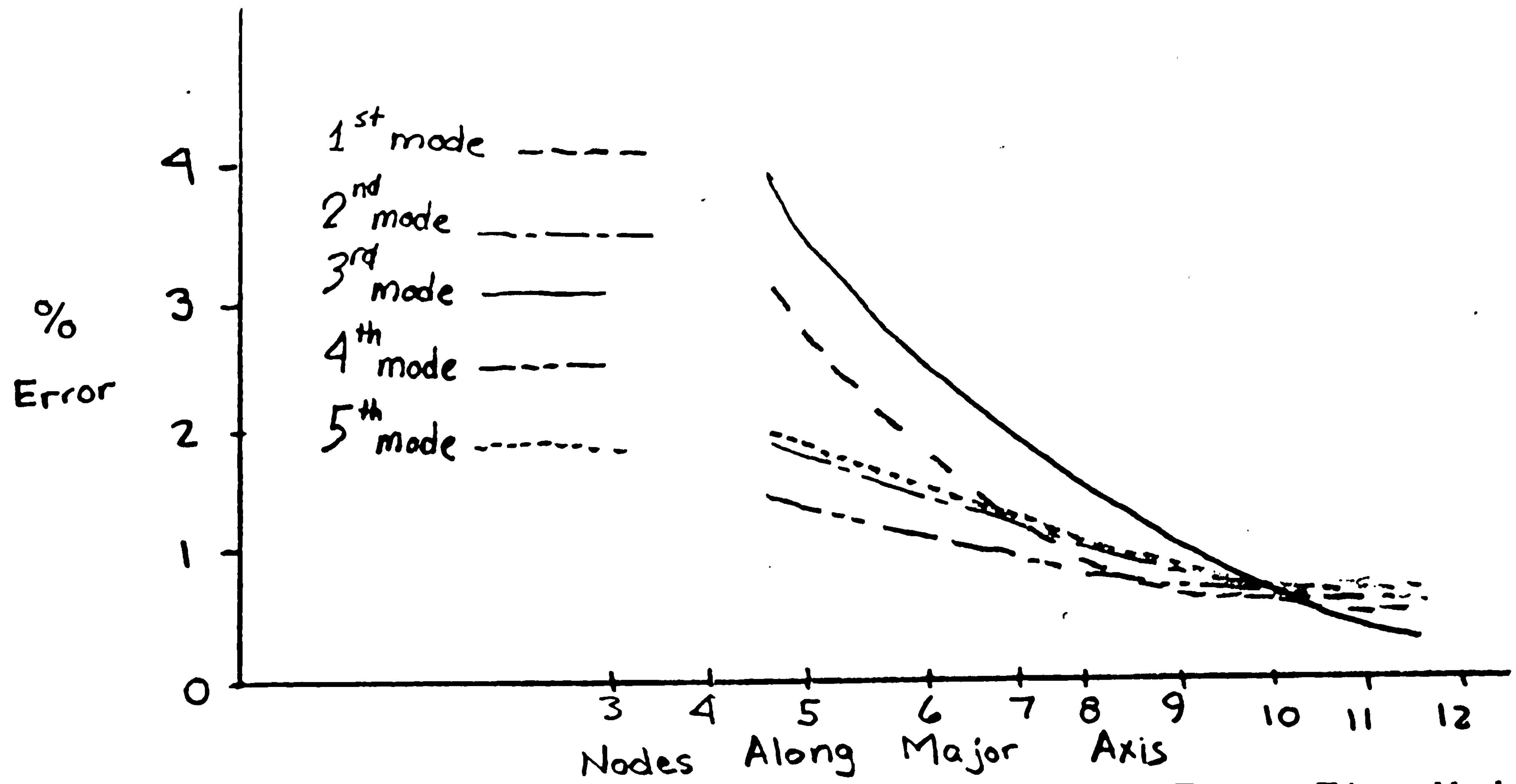


Figure 7. Percentage Error vs. Nodal Density for First Five Modes

Figure 7 shows the agreement of the first five modes for various numbers of nodes along each major axis. At five nodes along each axis (i.e. 125 nodes total), the error is between one and four percent. If the number of nodes along each major axis is increased to 12 (i.e. 1728 nodes total), error drops to about one half of one percent.

Spatial agreement analysis is good as long as the same natural frequencies are driving the respective systems. As mentioned before, the natural frequencies of the finite element space are slightly higher than the natural frequencies for the continuous space. Spatial distribution agreement at a given frequency is poor at higher frequencies due to the shift mentioned above. For instance, a driving frequency of 146.9 Hz excites the seventh mode (0,1,1) of the continuous solution, but the sixth mode (2,0,0) of the finite element solution. So in comparing the spatial distribution agreement, care needs to be taken to ensure that one is comparing the same mode shapes, rather than the same driving frequency.

IV. A. 4. Frequency Response Agreement

Frequency response is the response of acoustic pressure at an arbitrarily chosen point in space, usually in a corner where pressure is high, to a source emiting a tone of increasing frequency. The program can be used to compute a frequency response, but it is a bit cumbersome for it to do so. In order to compute the acoustic pressure at any one node, the program must solve for the pressure at every node. Furthermore, every change of frequency involves a completely new matrix problem, not just a resolving of a decomposed matrix with a new right hand side vector. So a model should not be loaded with more nodes than necessary, and a preliminary run with a much coarser mesh may be warranted to determine the approximate location of the natural frequencies and the general nature of the solution.

There appears to be excellent agreement in frequency response for a mode number of the same number of nodes along a major axis. Thus, for good agreement up through the fifth natural frequency there should be at least 5 nodes along each major axis of the model. A model with 12 nodes along each major axis, good agreement can be expected up through the first twelve natural frequencies.

Frequency response agreement appears to hold through a change of absorption. Error in this case is the difference in pressure (in decibels) at one particular node computed by the closed form solution minus the pressure computed by the finite element method solution, divided by the pressure of the closed form solution, averaged over every frequency tested, times 100. Error between the solutions decreases with increased absorption.

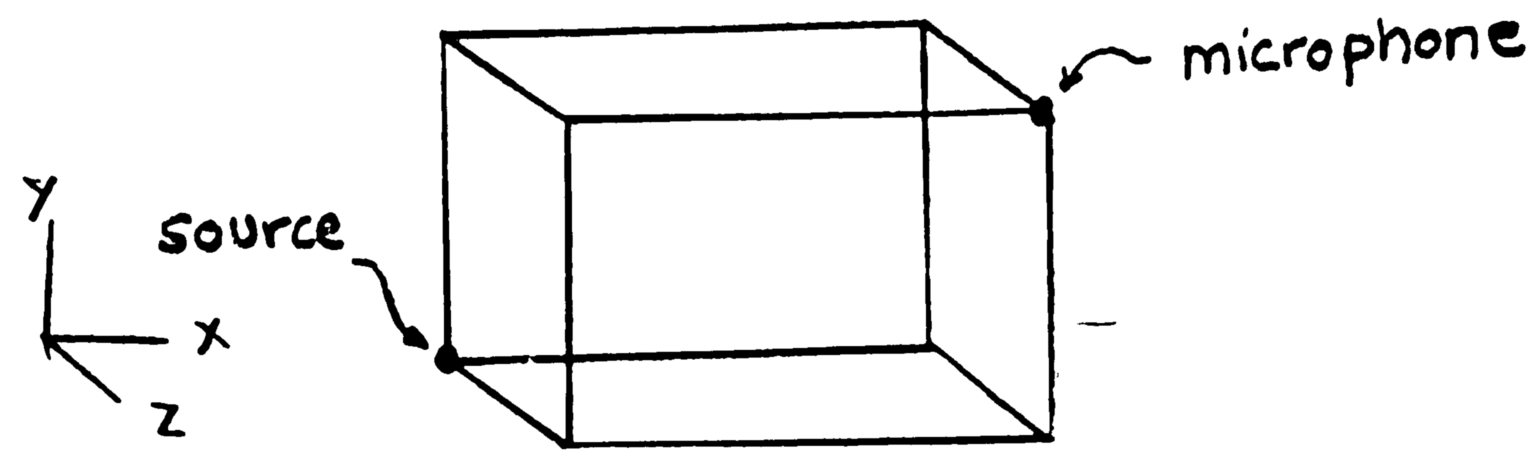


Illustration 4. Location of Microphone and Source in Rectangular Room

Figures 8 and 9 show a frequency response with respectively seven and twelve nodes along a major axis (i.e. 343 and 1728 nodes total, respectively). This frequency response is computed at the nodes in the opposite corner of the room from the driving source as pressure is always highest at the walls.

Agreement is good for both solutions at lower frequencies. Then Figure # shows that the finite element solution lags behind the closed form solution as frequency is increased, from about the midrange of the graph. By increasing the number of nodes, as Figure 9 shows, good agreement extends throughout the range of the graph.

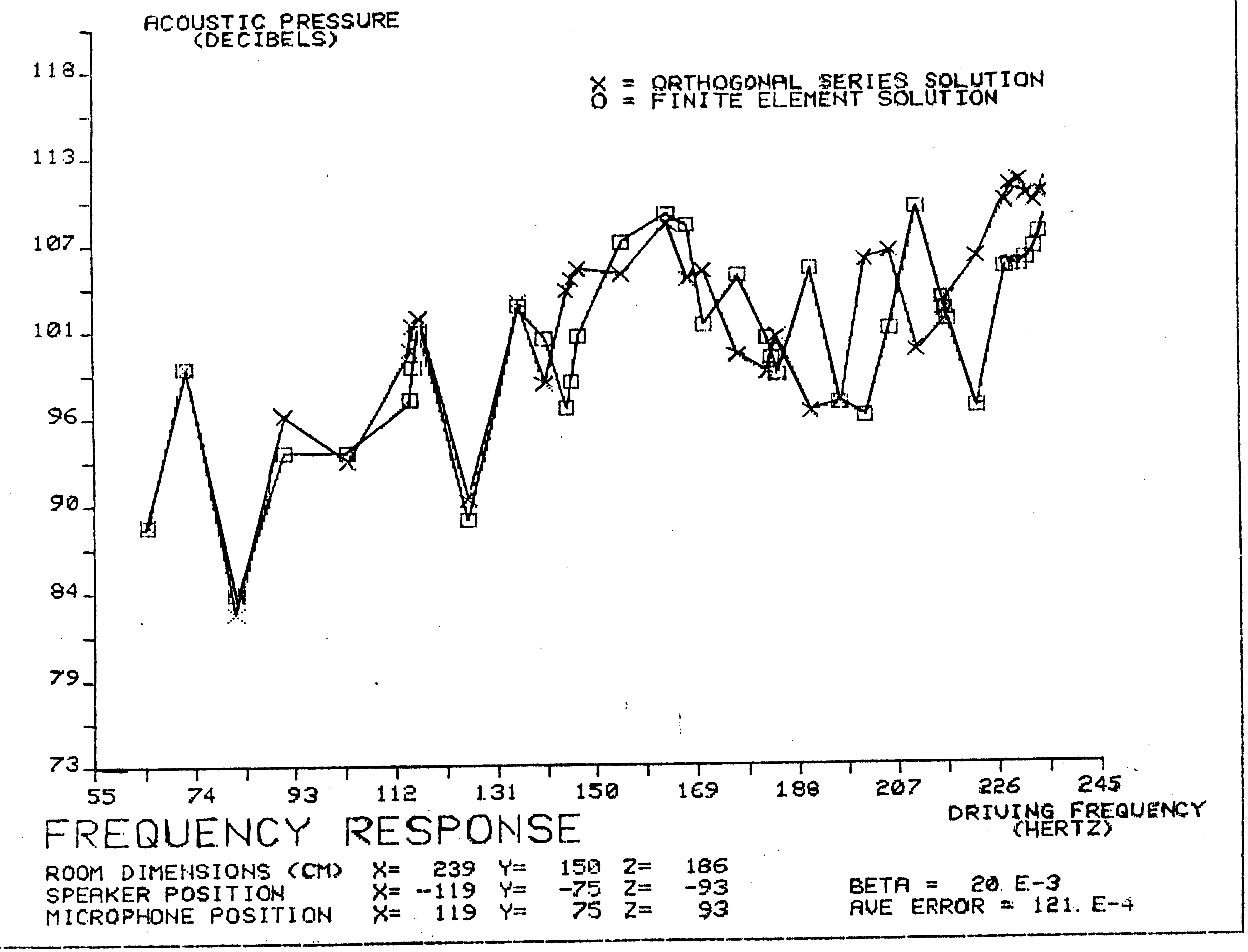


Figure . Frequency Response with 343 Nodes

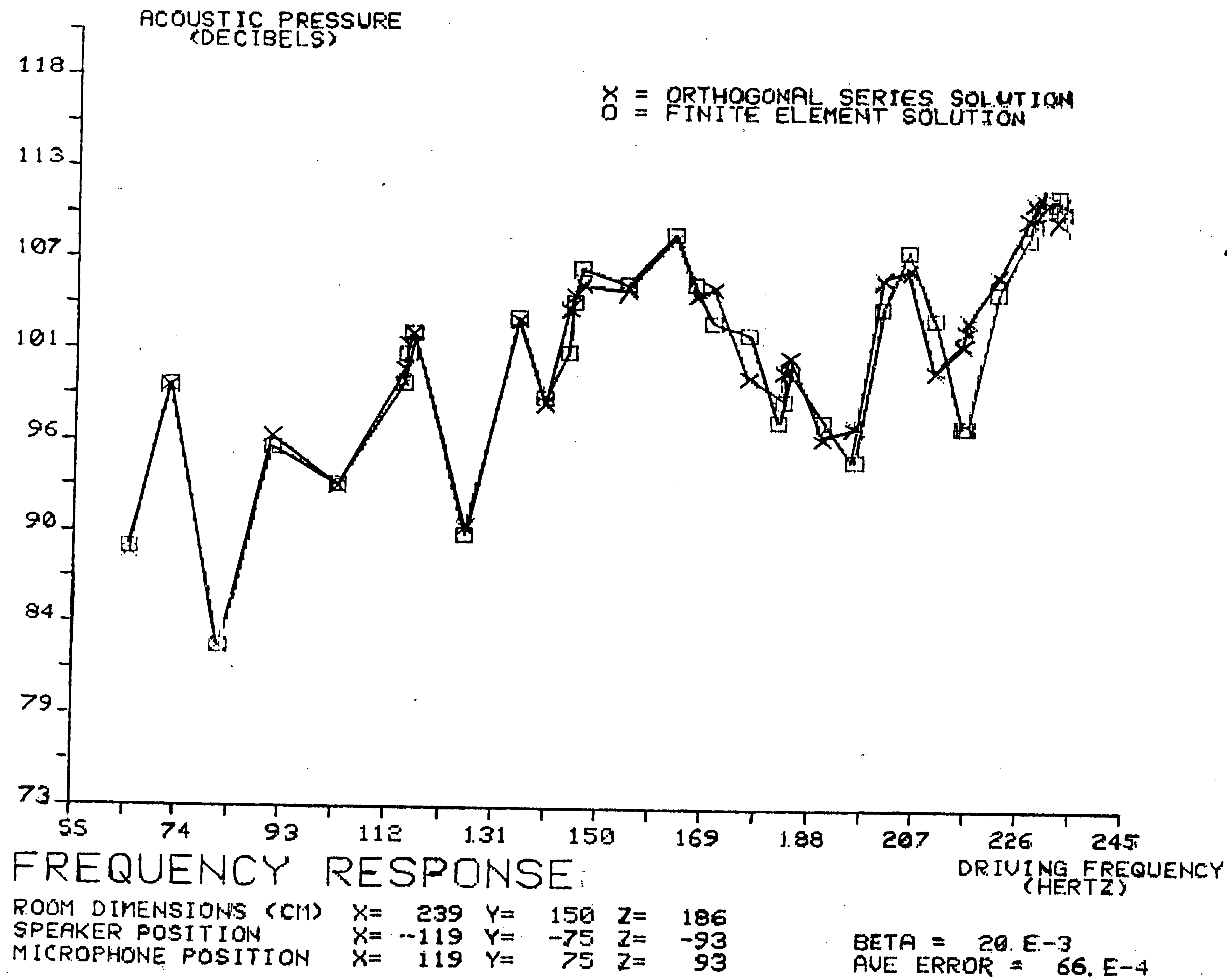


Figure . Frequency Response with 1728 Nodes

IV. A. 5. Absorption Variation

The table below shows the effect of increased absorption on frequency response for several meshes.

Table #. Percentage Error Between Frequency Response Curves

Nodes Along Axis	Coefficients of Absorption 0.005	0.020	0.035	0.050
5	5.0	2.0	1.3	0.9
7	3.2	1.2	0.4	0.0
9	2.4	1.0	0.4	0.0
12	1.8	0.7	0.6	0.4

The error shown is computed as mentioned in the above paragraph on frequency response. As just mentioned, the frequency reponse agreement appears to improve with increased absorption. This may be due to the fact that the peaks are not as sharp, but rather more smoothed out. The table shows that error is diminished by both increasing the number of nodes and by increasing the absorption.

One check of the validity of the finite element solution is whether it produces results that agree with the method of calculating damping coefficients used in physical experiments. The ratio of the bandwidth at the 3-dB down points from a peak to the frequency of that peak divided by the absorption is defined as the quality factor (Q) [5] and remains a constant for a given room with uniform wall absorption.

Figure 10 shows the frequency response for various values of absorption near the first and second natural frequencies. Absorption coefficients of 0.005 correspond to harder walls and show sharp peaks, while values of 0.050 are softer walls and a more gentle response. Figure # was generated by the finite element technique using 125 nodes. It shows a ratio of 1/74 for an absorption of 0.005, 2/74 for an absorption of 0.010, and 4/74 for an absorption of 0.020, roughly a linear correlation between the 3 dB downpoints ratio and the wall absorption. Dividing the 3 dB downpoints ratio by the absorption for all three curves near the first natural frequency yields a constant of aproximately 2.703. Near the second natural frequency, the ratio is 1.3/95, 2.6/95, and 5.2/95 for absorptions of 0.005, 0.010, and 0.020, respectively, which corresponds to a constant equal to 2.737. Hence, our agreement apppears to be confirmed, with less than 1 percent error.

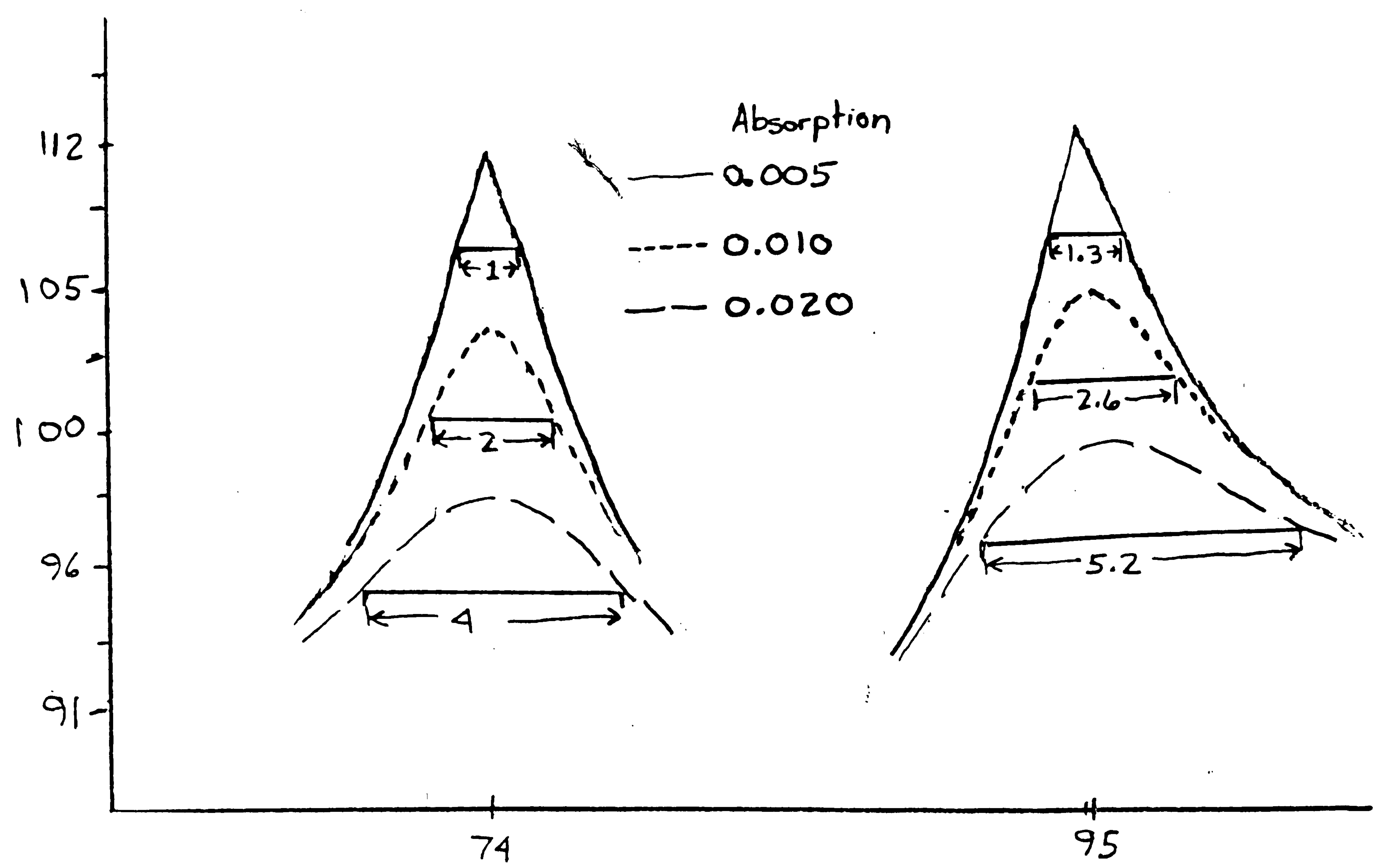


Figure 10. Frequency Response for Various Absorptions

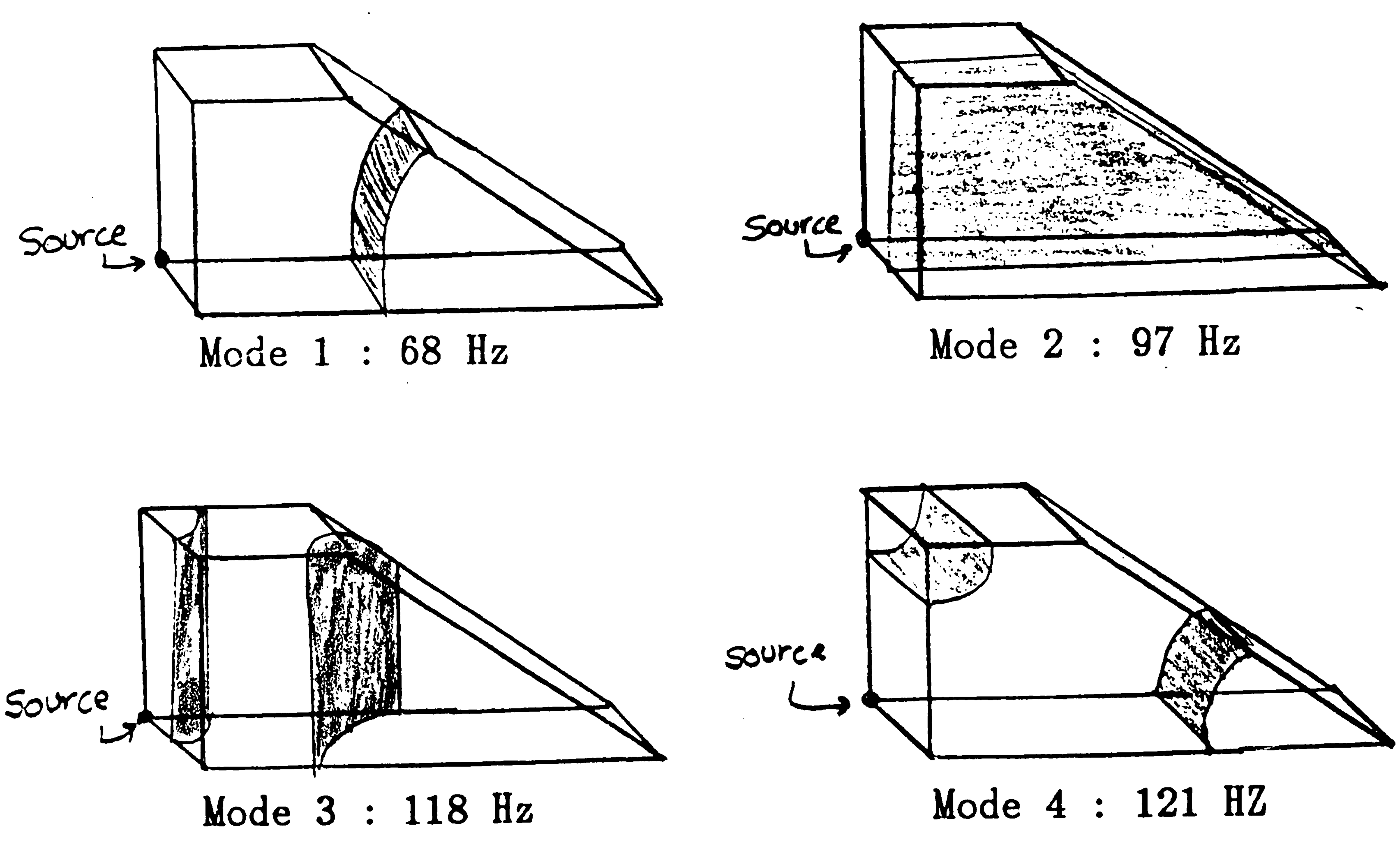
IV. B. MODIFICATIONS ON ROOM CHARACTERISTICS

IV. B. 1. Trapezoidal Room

The finite element solution can be applied to any shape. A room of the same depth, height, and volume but with the wall at x=1.865 tilted in at an angle of 45° was chosen because it varied only slightly from the rectangular enclosure, and differences would occur on a smaller scale that could be related back to the rectangular room.

In order to examine the model more efficiently, one should first find the natural frequencies from the Helmholtz Equation with the finite element method. Then the finite element method may be applied to determine the actual room reponse with a point source set at one of these frequencies, since the response at these frequencies show the most interesting information.

The following illustrations show the spatial distribution at some natural frequencies for this room.



Illust 5. Mode Shapes and Natural Frequencies of the Trapezoidal Room

IV. B. 2. Different Wall Absorbtion Coefficients Assigned to Different Walls or Parts of a Wall

Another important test for the finite element solution is whether it is actually responding the way it should for a change of wall absorption. The finite element solution cannot indicate mode shape (e.g., (1,0,0) or (0,0,1) or (1,1,0)). For example, the results will tell which mode is first, second, or fifth, but not that the first mode corresponds to (1,0,0) of the orthogonal series solution or that (1,1,0) is the fifth mode. However, this information is not necessary to the solution process, although the researchers can inspect the results and assign this correlation, if they desire. On the other hand, the finite element technique still accounts for changes in wall absorption by incorporating the increased or decreased value into the surface integrals.

In this case, all one need do is change the value of beta on the two respective surfaces. For instance, if the absorption on both y-z planes at each end of the x-axis is increased, it could be expected that there would be a major damping effect on the mode shapes which have a component in the x-direction, that is l is greater than 0 for (l,m,n) in the orthogonal series notation. Other modes will not be affected as much.

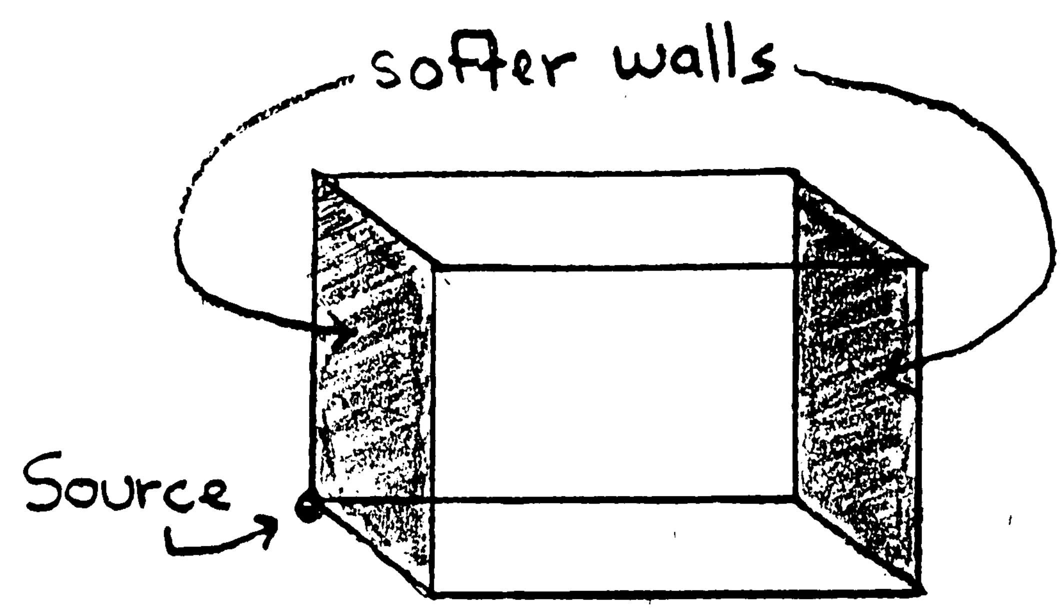


Illustration 6. Room with Two Soft Walls

The finite element solution does indeed respond just as expected, which demonstrates the strength of this model. In the model below, the wall absorption on the four walls that parallel the x-axis is set at 0.005 and the absorption on the two walls perpendicular to the x-axis varies.

Table 3. Acoustic Pressure Level (dB) with Increased Absorption on Two Walls

		Absorption on Walls Perpendicular to X-Axis		
Mode	Shape	0.005	0.010	0.050
1	1,0,0	108	108	99
2	0,0,1	98	98	96
3	0,1,0	101	101	98
4	1,1,0	113	112	104
5	1,0,1	111	111	104

As Table 3 indicates, the peaks for the first (1,0,0), fourth (1,0,1), and fifth (1,1,0) mode are damped from 7 to 9 decibels by increasing the absorption from 0.005 to 0.050 on the two walls which are hit by waves traveling along the x-axis. The values for the second (0,0,1) and third (0,1,0) modes dropped only 2 to 3 decibels, as expected, since these modes correspond to cases where no waves are traveling along the x-axis. This effect is significant, because although the solution has not accounted for mode shape, it still follows the expected pattern.

IV. B. 3. Several Sources at the Same Frequency and Phase

The model does handle two sources in the room at the same frequency. Two sources were placed on the $x = y = 0$ line, one at $z = 0$ and the other at $z = Z_L = 1.865$ m.

It shows that although the room has two speakers, both of them in corners, at a given source strength, the acoustic pressure near the walls at $x = 0$ and $x = X_L = 2.396$ m. becomes only 6 dB louder, increasing from 107 dB to 113 dB, than just one of them alone. This is in near agreement with Yerges' book [32] which mentions an approximate 3 dB increase when doubling the source.

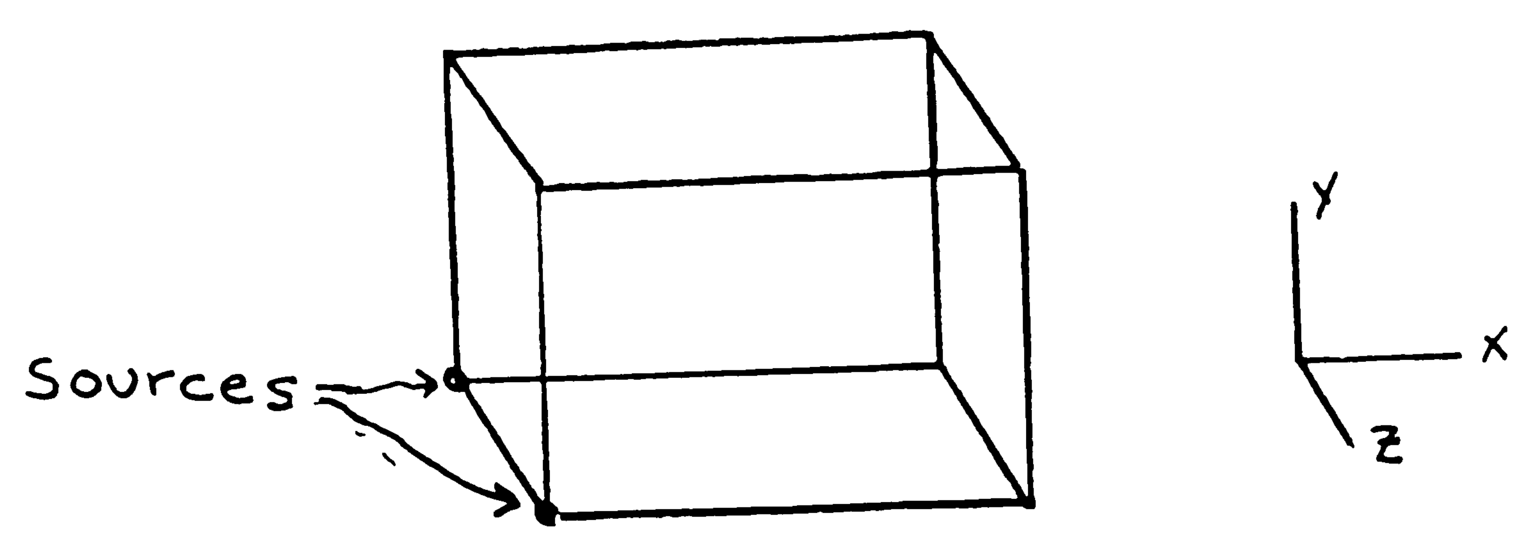


Illustration 7. Two Sources in the Rectangular Room

IV. C. COMPUTER TIME REQUIREMENTS

The amount of time required to solve a mesh increases with the size and set-up of the mesh. The order of the matrix is the same as the number of nodes used in the model. The bandwidth of the finite element matrix required by any one element is one plus the maximum global node number on that element minus the minimum node number on that element. For instance, if an element is made up of nodes numbered 26, 27, 32, 31, 51, 52, 57, and 56, the bandwidth of the finite element matrix required by that element will be 1 + 57 - 26 = 32. In order to minimize the amount of storage space required by the computer, the bandwidth of the matrix should be kept to a minimum by setting up the mesh such that for all the elements, each element is connected by nodes from as small a range of node numbers as possible.

The time required for the rectangular room example is tabulated below for different meshes on a VAX 11/780 computer.

Table 4. Analysis Time Required for Various Meshes

Number of Nodes	Time (sec)
60	1
125	4
216	9
343	31
512	100
729	321

The time required for solving the orthogonal series solution is about 3 seconds for 100 points (nodes), 25 seconds for 729 nodes, and about 80 seconds for 1728 nodes.

IV. D. MESH GENERATION

The only difficult task left to the user is to come up with the finite element mesh of nodes, elements, driving nodes and frequency, and absorption surfaces. To do so, determine the nodal locations in x-y-z space, and determine the elements, that is which nodes make up which elements, keeping the numbering of the sides consistent with the numbering of the nodes.

This study showed that it is very easy to write a mesh generating program for the block and trapezoid. More complicated shapes can be broken down to simpler ones. In general, it is easier to write a program to generate the mesh, rather than to write the mesh by hand. Figure 6 is a flow chart for mesh generation of the rectangular room.

In addition, it may be possible to interface existing three dimensional mesh generating CAD/CAM packages with the Acoustics System. Most are sophisticated enough to divide a volume into points and elements and can even allow an input of a source onto a node. This information can be translated with a translating program to give the acoustic mesh, driving node, and driving frequency. One may, however, encounter serious difficulties in assigning absorption surfaces if the mesh generating system cannot assign a value to a surface of an element.

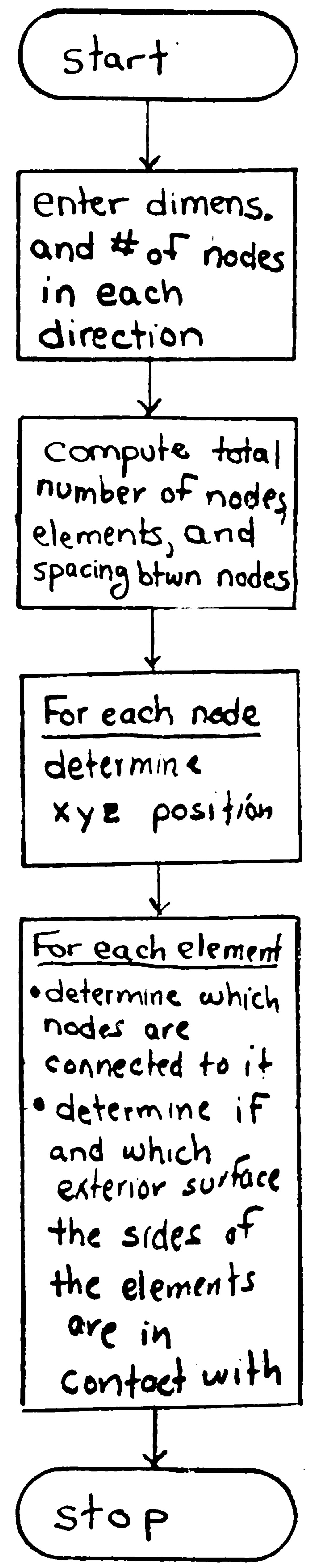


Figure 11. Flowchart for Rectangular Room Mesh Generator

Table 5. Sample Node and Element Mesh

Number of nodes: 27 Number of Elements: 8 Number of Surfaces: 2

Node Locations

Node	x	y	z
1	0.0	0.0	0.0
2	0.0	0.0	0.5
3	0.0	0.0	1.0
4	0.0	0.5	0.0
5	0.0	0.5	0.5
.	.	.	.
.	.	.	.
12	0.5	0.0	1.0
.	.	.	.
.	.	.	.
26	1.0	1.0	0.5
27	1.0	1.0	1.0

Nodes and Sides on the Element

Element	N1	N2	N3	N4	N5	N6	N7	N8	S1	S2	S3	S4	S5	S6
1	1	2	4	3	10	11	14	13	2	0	0	0	1	1
2	2	3	6	5	11	12	15	14	2	1	0	0	0	1
3	4	5	8	7	13	14	17	16	2	0	1	0	1	0
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
7	13	14	17	16	22	23	26	25	0	0	1	2	1	0
8	14	15	18	17	23	24	27	26	0	1	1	2	0	0

Many mesh generating systems permit assigning a value of pressure to an element face. This feature could be used to assign the absorption to an acoustic element face, as long as the database stores the pressure data on the <u>side of the element</u>, but this feature could not be used if it translates the pressure data into <u>nodal</u> loads before storing into the database. Without this feature, the mesh information becomes too complicated to manipulate and the user will find it impossible to decide on his own where to place the absorption surfaces. Furthermore, a general translation file could not be expected to handle this kind of information. In this case, the user might consider breaking the model into simpler geometric parts and writing his own mesh generating input program.

V. CONCLUSIONS

V. A. VALIDITY OF THE FINITE ELEMENT MODEL

The finite element solution compares well with the closed form solution for a rectangular room with a point source and homogeneous absorption material on all walls over the low range of frequencies and for several absorption coefficients. However, at higher frequencies corresponding to higher eigenvalues, the low mesh density introduces error similar to an increase in stiffness. Effective use of this solution for spatial distribution and frequency response analysis for forced vibration requires appropriate finite element mesh density, adequate computer resources and computer graphics to interpret the results.

V. B. ELEMENT SIZE

In general, a comparision of the natural frequencies derived by the finite element method show that as the element size is decreased and thereby more elements are included in a given space, the values approach those derived by the closed form solution. In general, for a given frequency, the spatial distribution will approach that for the closed form solution as more elements are added. The following graph shows the accuracy of the solution with an increasing number of nodes along a major axis of the model.

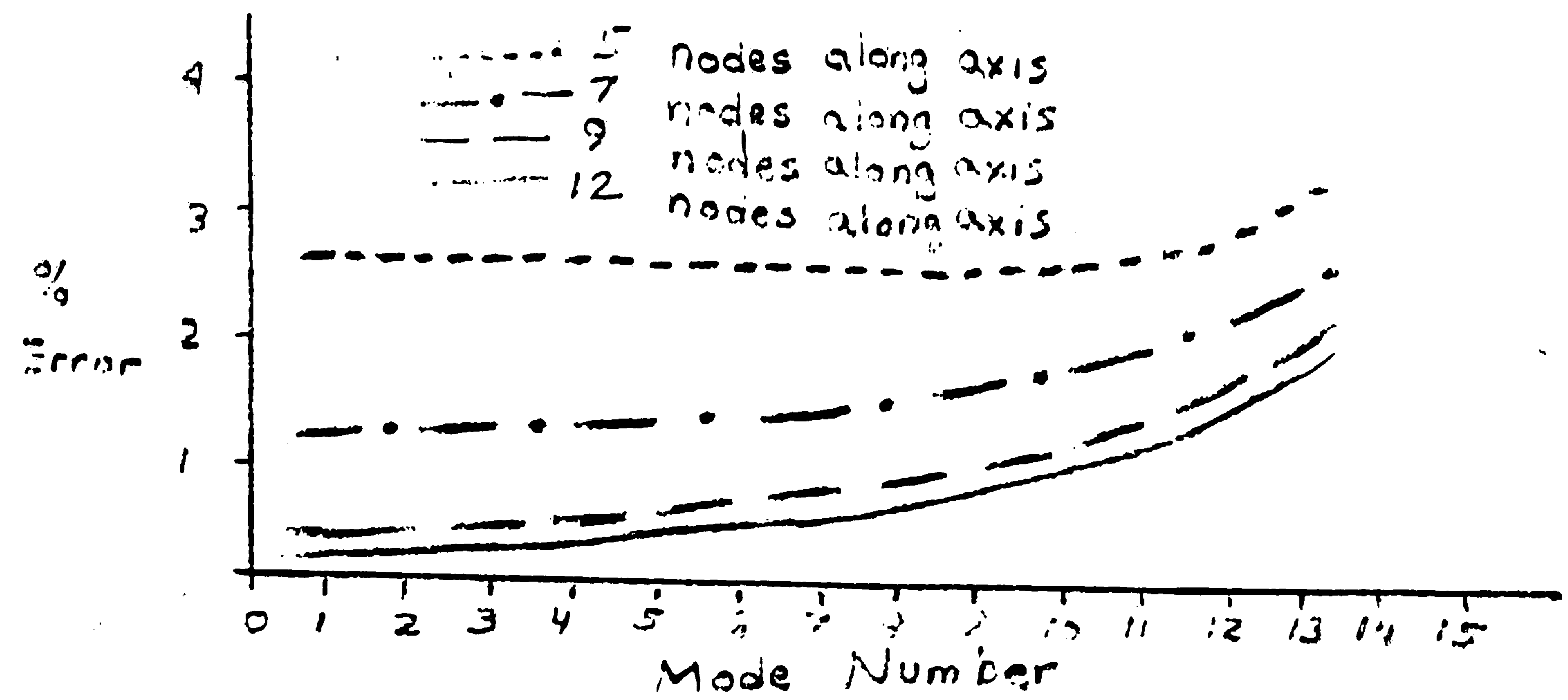


Figure 12. % Error vs. Mode Number for Increasing Mesh Densities

In Figure 12, the average spatial percentage error is computed at each natural frequency (or mode) as the difference between the closed form model pressure (in decibels) and the finite element model pressure, divided by the closed form solution for pressure, times 100. This graph shows that the general trend is that the error increases at higher frequency, that is to say higher modes, for any mesh, and that the more elements in a given mesh, the less the error.

The following rules appear to govern mesh density:

1. Enough nodes should be included in a model to display an expected mode shape. This means there should be at least five nodes to display one half of a sine wave, or nine nodes to display one complete sine wave.
2. The model must have enough nodes in any direction for the mode shape to be visible or computed with reasonable accuracy.
3. More elements improve the accuracy of the spatial distribution.
4. More elements slow down the inflation of natural frequencies process in a frequency response analysis.
5. More elements are more expensive to run.

V. C. LIMITATIONS

V. C. 1. Solving a Large Matrix

A major problem with the finite element method in general is that it requires a large matrix. In three dimensional models, this hardship is increased as the number of nodes needed to solve a proble increases dramatically. In the case of wall damping with a point source, it becomes even more acute, since this introduces complex numbers and the storage size doubles on account of that. Fortunately, the problem involves a symmetric matrix, and symmetric storage for matrices can be utilized.

V. C. 2. Stiffness

The most serious limitation is the upward shift of the natural frequencies from that of continuous space. The effect increases with natural frequency, i.e. the higher the continuous natural frequency, the greater the shift upwards. However, at higher frequencies, the modes are closer together and overlap. Thus, mode shapes may not be clearly seen at high frequencies anyway. This limitation may be easily corrected by using a driving frequency slightly inflated from the true physical driving frequency in the analysis of the problem.

V. C. 3. Application to Various Enclosures

This solution can be applied to any room or any reverberant space, but that space will be subject to the approximation of curved boundaries with straight line, due to the order of the interpolating/coordinate mapping function.

For the accuracy of the information produced, this approximation is not considered very significant, especially in light of the economy of running this method.

Walls, or even parts of a wall, can be assigned different absorptions, which vary greatly from one another. For instance, if a carpet is covering part of a floor, the absorption of the carpet can be specified in the model to be much greater than that of the hard floor, and similar provisions for windows can be made. Obstructions, such as balconies or supports can also be included in the model, as can chairs and aisles.

Thus, the model does have a wide range of applicability and can be used to model any reverberant space driven by one or more point sources at a given frequency.

V. C. 4. Study of Only One Mode Shape

Another limitation of the finite element solution with regard to frequency response is that the contribution of one mode, or the frequency response of one mode cannot be measured. This is because the finite element method arrives at the solution from matrix solving, not by contribution of the mode shape to a sum. The mode shapes can be obtained for study, however, by solving for the Eigenfunctions of the Helmholtz Equation.

V. D. RECOMMENDATIONS FOR FURTHER STUDY

The validity of the finite element method should be studied by comparing it with the closed form solutions for other simple shapes, such as cylinders and spheres, and then compare those results to physical experiments for those shapes.

A very useful modification to this program would be the inclusion of sources distributed over a wall so that the response of the enclosure to the motion of a wall in vibration could be analyzed. This extension would increase the model's usefulness in studying noise due to structurally induced sources.

Higher order interpolation and integration functions might be used in the finite element derivation. Higher interpolation functions generally involve more nodes, or more information at each node, and can be used to model curved surfaces, such as the Petyt twenty node element of reference [17]. Higher integration functions involve mapping at more Gauss points in r-s-t space, to get a higher order curve fit. The accuracy of the fit is governed by the type of interpolation function used. It would be wasteful to use an algorithm which will perfectly fit up to a sixth order curve, when the integrand is only second order. These modifications would result in slightly improved accuracy, but would also increase the cost. However, fewer elements would be needed to model a given space, they could be used to model curved surfaces, and the acoustic pressure between the nodes would be modeled more as it should, as the response is curved over space.

Many existing finite element packages have enough versitility to handle new and different elements and problems. NASTRAN has the capability to run acoustic cavity elements for the determination of natural frequencies and also the expandibility needed so that a reverberant room with wall damping and a point source could be analyzed. Alternatively, a package should be able to include some Helmholtz matix terms, and/or have the versitility to permit borrowing some other matrix terms, such as the mass matrix from solid mechanics.

Air damping could be incorporated into this model to increase its accuracy, again at an increased computer cost for minimal results.

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Appendix I

User's Manual for
A Finite Element System for Acoustics

User's Manual for
A Finite Element System for Acoustics
(ACOSYS.LU)

I. INTRODUCTION

I. A. PURPOSE

This system was developed in order to manipulate a node and element mesh of any complex shaped enclosure, to process that mesh for acoustic pressure or natural frequencies, and to display the results on a VS 11 color computer graphics terminal.

I. B. INTERNAL STORAGE

Storage of the information in the system is in several arrays.

IELEM is an array of the number of elements by eight, which contains the number of the eight global nodes for an element. The numbering of the global nodes must stay geometrically consistent with the numbering of the local nodes on the local elements which is

```
  6 -- 5
 /|   /|        s
2 + -- 1 |      |
| |   | |   in  |__  r  .
| 7 -- + 8     /
|/    |/      t
3  -- 4
```

IELEM(IEL,IND) contains the global node number of the node corresponding to the IND th local node of the IEL th global element.

IBETA is and array of the number of elements by six, which contains the number of the exterior side.a side of the element is in contact with. If the side of the element is in contact with another element instead of an exterior surface, a zero is recorded in IBETA.

All of one wall can be given one absorption value, or several walls could be lumped together as one continuous surface. In fact, even just parts of a wall could be one absorption, differing from the absorption on the rest of that wall. The smallest area which can be assigned an independent absorption is the side of an element. Absorption coefficiensts can be assigned to different surfaces by the designer, and can be varied during the course of a session. However, surfaces are defined in the mesh generating program, and cannot be changed in the system. Hence, a ceiling may be called one surface, while the four walls and floor could be lumped together as a second surface. During the course of the session, the absorption on the ceiling can be varied by the designer. He may even choose to assign the same value to the ceiling as the other walls. But, unless defined as a seperate surface in the mesh generating program, the floor could not be independently assigned. It is therefore important to determine all of the surfaces which might have a different absorption in the mesh generating program, because it is to late to do it in the system.

The numbering of a side must stay geometrically consistent with the numbering of the nodes. The following ordering rule was used in this Acoustic System:

```
   6  --  5
  /|     /|        s                     6 4
 2 + -- 1 |        |                     |/
 | |    | |   in   |__  r  with sides 2 --+-- 5
 | 7 -- + 8       /                     /|
 |/     |/       t                     1 3
 3  --  4
```

On each side, the nodes are ordered counter-clockwise.

BETAS is a one dimensional array of maximum length twenty, in which the values of the different absorption surfaces can be stored.

BETA(IWL) is the absorption of the IWL th exterior walll surface.

R is an array of the number of nodes by three which contains the xyz space location of the nodes. Nodes are numbered consecutively starting with one.

R(IND,1) contains the x position of the IND th node.
R(IND,2) contains the y position of the IND th node.
R(IND,3) contains the z position of the IND th node.

Models should be built in positive xyz space as close to the origin as possible. For good (automatic) scaling, the last node should be on aan opposite side from the first node in the objects long dimension.

II. DESCRIPTION OF FUNCTIONS
II. A. ENTERING THE SYSTEM

The system is stored in the program file ACOSYS.LU. The user types

RUN ACOSYS <RETURN>

The system responds by initializing variables and printing a title block to the screen to acknowledge to the users that they are entering the Acoustic System.

FINITE ELEMENT MODELING
OF ACOUSTIC ENCLOSURES

Written by:
Larry Sabo

Under the Direction of:
Dr. J. Ochs

Winter, 1985-86

II. B. LOAD MESH MENU

This system is menu driven by selecting the first letter of the desired option in either upper or lower case. The first menu a user sees is not the MAIN menu, but the LOAD MESH menu, which allows a user to specify a file with an acoustic finite element mesh.

CURRENT READ FILE IS UNINITIALIZED

H_ELP
N_ODE FILE
M_AIN MENU

A mesh need not be loaded at this time as the program has initialized all the variables, but a mesh written previously to entering the system must be loaded in at some point before the computer can display old results or compute new results. The system currently has no way to write a mesh, so this task must be done outside the Acoustic System.

II. B. 1. HELP

HELP gives the user information on how to set up and format a node file. Upon selecting HELP, the user is given the following information:

FILES MUST CONTAIN NODAL INFORMATION AS FOLLOWS:

# of Nodes, Elements, and Absorption Surfaces
Inode, x-pos, y-pos, z-pos, (Inode=1, # of Nodes)
Iel, IELEM(Iel,Ind),(Ind=1, 8), IBETA(Iel,Isd),(Isd=1,8),(Iel=1,# Nodes)
(Following may be repeated as often as desired up to 38 times)
Driving Node Number, Driving Frequency
Absorption coefficients for Surfaces 1 through Max of 20 (rows of 10)
Nodal Pressures for nodes from 1 through # Nodes, (rows of 12)

NOTE:
It is important to keep consistent with coordinating the numbering of the sides with the node numbers.

Press <RETURN>

II. B. 2. NODE FILE

Under this option, a file containing the nodal information in the format specified under HELP is loaded into the system. Upon selecting NODE FILE, a user is querried for the name of an existing nodal file.

ENTER FILE NAME

The user enters the file name and presses the <RETURN> bar. (A user could enter "SYS$INPUT" here and enter the nodal data by hand in the appropriate format, but that would be very impractical due to the size of the data.) The system loads this information, and tells the user the highest node number.

HIGHEST NODE NUMBER = max node no.
PRESS <SPACE BAR>

Upon pressing the space bar, the system enters the MAIN MENU. When the user returns to the LOAD NODE FILE MENU, the line preceeding the menu will contain the current READ FILE name:

CURRENT READ FILE IS filename

II. B. 3. MAIN MENU

If no action is desired in the LOAD NODE FILE, or after the HELP option has been viewed, the user is returned to the MAIN MENU.

II. C. MAIN MENU

All functions can be accessed from the MAIN MENU, and it is to this menu a user is returned when complete with a function. The Menu permits the user to select the action desired, whether it be loading another mesh file, getting results out of the mesh file, changing variables, processing the model for either acoustic pressure at the nodes or for the natural frequencies, writing results to a file, or terminating the system.

LOAD NODAL FILE
GET OLD RESULTS
CHANGE VARIABLES
SUBMIT MODEL TO BE PROCESSED
NATURAL FREQUENCIES (HARD WALL)
WRITE TO A FILE
EXIT FROM ACOUSTIC SYSTEM

II. D. GET RESULTS MENU

The program is capable of getting results from a mesh file if the results are stored after the mesh information. This precaution ensures that the results do not get separated from the mesh from which they were created. Thus, a user can make several runs in advance of entering the acoustic system and call up the spatial distribution at one of those driving frequencies or absorption coefficients, or if all the runs have the same absorption coefficient, to display the frequency response at one node. Or, he could recall the results from a previous Acoustic System session. Upon selecting this function, a user will see another menu:

SPATIAL DISTRIBUTION
FREQUENCY RESPONSE
MAIN MENU

II. D. 1. SPATIAL DISTRIBUTION

The SPATIAL DISTRIBUTION option is to show the user a spatial distribution of a run stored in the current read file. See SPATIAL DISTIBUTION for more information on this. Upon selecting to see a spatial distribution, the system will ask the user which run in the file is intended.

ENTER RUN NUMBER

Therfore, users must keep track of what is in the file, and where it is. Otherwise, it is suggested that the user find it by a FREQUENCY RESPONSE, or by trial and error. The user enter an integer value of the run of interest, and presses the return key.

The system will load the driving node and frequency, the absorptions corresponding to that run, and the acoustic pressure at all the nodes, and display a SPATIAL DISTIBUTION. (See SPATIAL DISTRIBUTION.)

II. D. 2. FREQUENCY RESPONSE

Selecting to see a FREQUENCY RESPONSE, the User must tell the system at which node it should generate this graph.

ENTER NODE NO.

The designer enters an integer value of the node at which the frequency response is desired. The system will state how many runs were used to generate the curve. The user selects DONE to return to the GET RESULTS MENU.

RUNS
D_ONE

See FREQUENCY RESPONSE for more information.

II. D. 3. MAIN MENU

If no action is desired, or after a spatial distribution or frequency response has been viewed, the user returns to the MAIN MENU via this option.

II. E. CHANGE VARIABLES MENU

The system allows the user to change variables in the CHANGE VARIABLES menu and to make another run. Thus, a designer interested in the response at another frequency or the effect of a different absorption may have on the frequency or spatial response. The driving node or frequency may be changed. (Currently there is no provision in the program for handling multiple sources at the same frequency. And the finite element model presented in this paper does not account for multiple sources at different frequencies.) Source strength, density, temperature, and reference pressure may also be changed. The computer also has a graphics controls key for selection of the desired graphics format.

At the start, the menu would look like this:

```
F_REQUENCY DRIVING = 0.00
N_ODE DRIVING = 0
B_ETA ON  EXISTING WALL SURFACES
 SURFACE NO. 1 ABSORPTION = 0.00000
 SURFACE NO. 2 ABSORPTION = 0.00000
  .   .   .   .   .   .   .   .
  .   .   .   .   .   .   .   .
 SURFACE NO. (last) ABSORPTION = 0.00000
P_REF = 2.0E-5
R_HO  = 1.21
T_EMP = 24.5
Q_NOT = 1.7395E-3
I_NKJET = 0
M_AIN MENU
```

II. E. 1. FREQUENCY DRIVING

To change the driving frequency, the user selects this option. The system responds by asking for a new value.

ENTER NEW DRIVING FREQUENCY

The user enters a new value of driving frequency and presses the return key.

II. E. 2. NODE DRIVING

To change the driving node, the user selects this option. The system responds by asking for a new value.

ENTER NEW DRIVING NODE

The user enters a new value of driving node and presses the return key.

II. E. 3. WALL ABSORPTION

To change the wall absorption, the user selects this option. The system responds by asking for which surface.

ENTER SURFACE NO

The user enters a new surface number and presses the return key. The system then asks for the new value of absorption.

ENTER ABSORPTION

The user enters the new absorption coefficient and presses the return key.

II. E. 4. INKJET

This option reverses black and white colors on the screen to make for better inkjet plotting. For normal screen viewing on a black background, INKJET should be set equal to zero. For inkjet plotting on a white background, INKJET should be set equal to 1.

II. E. 5. OTHER VARIABLES

Other variables which may be altered are reference pressure, density, temperature, and source strength. The user selects the variable he or she wishes to change and the computer responds with a message that this change will not be recorded into the database file, although it will be used in all further computations.

WARNING: ALTHOUGH NEXT RUNS WILL USE THIS VALUE
NO RECORD OF THIS CHANGE WILL BE STORED IN THE FILES

ENTER NEW (variable)

The user enters the new variable and presses the return key.

II. F. SOLVING FOR PRESSURE OR NATURAL FREQUENCIES

The program has the capability to process the model for the acoustic pressure in decibels with the conditions assigned in CHANGE VARIABLES and to then graphically display the results in a spatial distribution. (For more information on spatial distribution, see SPATIAL DISTRIBUTION.) Alternatively, the program could compute the natural frequencies for the hard wall model, using the IMSL subroutine EIGZST which solves the Generalized Eigenproblem, and list these frequencies to the screen. Upon selecting these options, the user is not given further menu options, but rather a confirmation that the model is currently being processed.

II. G. WRITE MENU

In the WRITE menu, the system allows a user to specify a new file name, so that information could be written to several files. Nodal data and acoustic pressure could be written in a format that can be recalled by the system at a later time, and displayed graphically in the GET OLD RESULTS menu. Natural frequencies can be written to a file so that the designer can have a hard copy of these important frequencies.

Upon selecting this function, the user has the following options:

CURRENT WRITE FILE IS UNINITIALIZED

S PECIFY A NEW WRITE FILE
N ODAL DATA - WRITE TO FILE
P RESSURE DATA - WRITE TO FILE
H ARD WALL NATURAL FREQUENCIES -WRITE TO FILE
M AIN MENU

II. G. 1. SPECIFY NEW WRITE FILE

This function is to specify a brand new file to write data to.

ENTER FILE NAME

The user enters a new filename and presses return. The system creates a new data file. The menu reappears, but this time, the preceeding line states:

CURRENT WRITE FILE IS filename

Note: If the WRITE FILE is specified as SYS$OUTPUT, all data will be displayed onto the screen. Note also that once another file is specified, the user cannot reactivate the first file. However, the user could append the second file to the first after the session is over.

It is recommended that the file names refer to shape and/or absorption, and that each file contain the same values of absorption for all surfaces so that frequency responses can be generated.

II. G. 2. DATA - WRITE TO FILE

These options write the respective data to the end of the file. NODE DATA option writes node, element, and exterior absorption surface mesh data to the file. PRESSURE DATA writes the driving node and frequency, the absorption surface coefficients, and the acoustice pressure in decibels to the bottom of the file.

In order to recall data to the system at a later time, data must be stored in a certain order. Nodal data must be stored first in the file, followed by as many sets of pressure data as are of interest to the designer up to 38 sets. Natural frequency data or another set of nodal data buried in the sets of pressure data will cause errors in GET OLD RESULTS.

It is recommended that natural frequencies be stored in a seperate file.

II. H. TERMINATION

Upon termination of the Acoustic System from the MAIN menu, the system clears all graphics left up on the screen and terminates the system, returning the user to the operating system.

II. I. SPATIAL DISTRIBUTION

The spatial distribution function allows a user to see the mode shape or spatial distribution at a given driving frequency. The display is three views of the model: top, front, and side in the usual drafting layout style, each view tilted by five degrees to allow the designer to look down the rows of nodes. In some models, some of these views may look meaningless or messy, but with three views, there is a better chance of seeing a view whichg does have some meaning. Each view is a display of node points in three dimensional space. The acoustic pressure is represented by the color of the node. The nodes are made visible in their color starting with the nodes at low pressure areas (so that nodal planes may be represented) and the nodes with increasingly higher pressure are made visible in their color. The process may be temporarily suspended by pressing the NO SCROLL key to examine the reverberant space as it is filling in increasingly higher presure levels. Pressing the NO SCROLL key a second time allows the computer to continue on filling in with the higher pressure levels. A color key, as well as information about the driving source and absorption is printed in the upper right hand corner of the picture. After a run has been displayed, the user can press "R" for REPEAT (an instant replay of the room filling up with colored dots) or be DONE with that display and return to the current menu.

II. J. FREQUENCY RESPONSE

The frequency response graphics produces a graph of the acoustic pressure in decibels versus the frequency in Hertz. The pressure level at a selected node will be displayed as a point on the graph at the associated frequency.

In order to see a meanigful frequency response, some care must be taken by the designer. The system does not check valuews of absorption from one run to the next. Fyrthermore, the system assumes that all data is stored in either increasing or decreasing order, and draws a line from point to point. Thus, if driving frequencies are stored out of order, the frequency response will show lines scribbling back and forth across the graph. Therefore, it is the responsibility of the designer to be sure that the absorption coefficients in a file are the same, and runs are stored in either increasing or decreasing order.

II. K. HARDCOPIES

Hardcopies of the graphics may be obtained by typing 'Control-Y' to interupt the program when the desired screen graphics, and then using one of the screen-to-paper algorithms. Hardcopies of the mesh, acoustic pressures, and/or natural frequencies can be obtained by writing that data to a file and printing that file from the operating system.

III. AN EXAMPLE SESSION

An example mesh file is stored in TRAP2S.LU. This is a mesh of a trapezoidal room with the tilted wall a different absorption from the rest of the room.

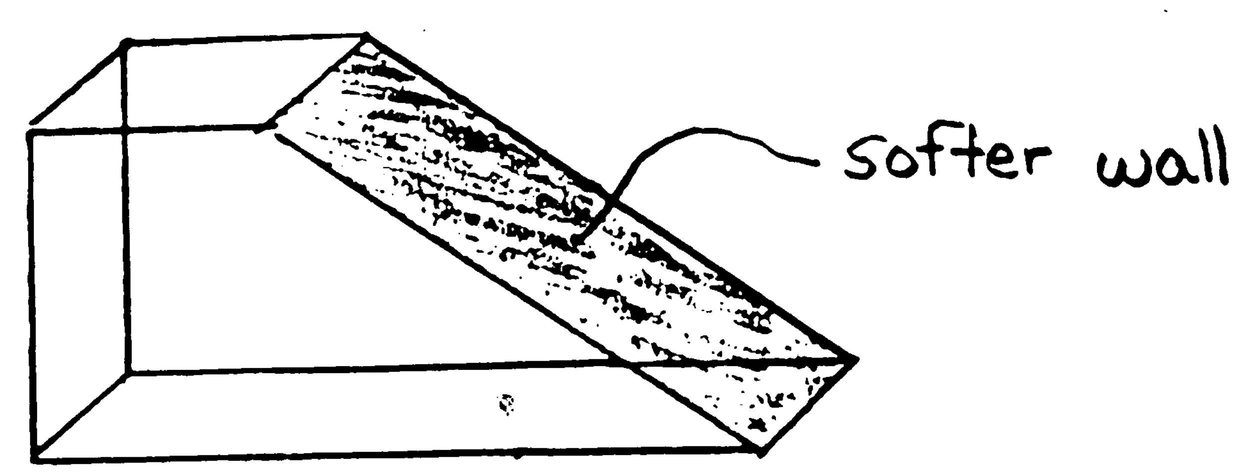

Figure A.I.1 Trapezoidal Room with Different Absorption on the Tilted Wall and Elemental Representation

Entering the system, the user types:

RUN ACOSYS

The system responds with its title block.

FINITE ELEMENT MODELING
OF ACOUSTIC ENCLOSURES

Written By:
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Under the Direction of:
Dr. J. Ochs

Winter, 1985-86

And the LOAD NODE FILE MENU.

CURRENT READ FILE IS UNINITIALIZED

HELP
NODE FILE
MAIN MENU

Pressing 'H' for HELP, the user sees the display of the format required for the nodal file.

FILES MUST CONTAIN NODAL INFORMATION AS FOLLOWS:

# of Nodes, Elements, and Absorption Surfaces
Inode, x-pos, y-pos, z-pos, (Inode=1, # of Nodes)
Iel, IELEM(Iel,Ind),(Ind=1, 8), IBETA(Iel,Isd),(Isd=1,8),(Iel=1,# Nodes)
(Following may be repeated as often as desired up to 38 times)
Driving Node Number, Driving Frequency
Absorption coefficients for Surfaces 1 through Max of 20 (rows of 10)
Nodal Pressures for nodes from 1 through # Nodes, (rows of 12)

NOTE:
It is important to keep consistent with coordinating
the numbering of the sides with the node numbers.

Press <RETURN>

Pressing the return key, the system displays the NODE FILE MENU again.

CURRENT READ FILE IS UNINITIALIZED

H_ELP
N_ODE FILE
M_AIN MENU

Typing 'N' for NODE FILE, the system asks the user for a file name.

ENTER FILE NAME

The user responds by loading in the name of an existing Nodal file.

TRAP2S.LU

The system tells the user how many nodes are in the mesh.

HIGHEST NODE NUMBER = 125
PRESS <SPACE BAR>

The user presses the space bar and the system displays the MAIN MENU.

L_OAD NODAL FILE
G_ET OLD RESULTS
C_HANGE VARIABLES
S_UBMIT MODEL TO BE PROCESSED

N_ATURAL FREQUENCIES (HARD WALL)
W_RITE TO A FILE
E_XIT FROM ACOUSTIC SYSTEM

The user selects 'G' to see what solutions are in the file. The system displays the GET OLD RESULTS MENU.

F_REQUENCY RESPONSE
S_PATIAL DISTRIBUTION
M_AIN MENU

The user selects 'F' for a FREQUENCY RESPONSE. The system asks the user to specify the node at which to generate the frequency response.

ENTER NODE NO

The user specifies the node and presses the return key.

101

The computer prints a small message that it is reading the file.
READING FILE ...

Afterwards it displays the frequency response and states how many sets of acoustic pressure data are stored in the file.

38 RUNS
D_ONE

The user presses 'D' when (s)he decides to move on. The OLD RESULTS MENU is displayed again.

F_REQUENCY RESPONSE
S_PATIAL DISTRIBUTION
M_AIN MENU

The user asks to see a SPATIAL DISTRIBUTION by pressing 'S'. The system responds by asking for the run number.

ENTER RUN NUMBER

The user chooses to see the response near the first mode and enters '5' and presses the return key.

5

The system recalls that run and shows a spatial distribution. Afterwards it asks the user if (s)he wants to see an instant replay.

R_EPEAT
D_ONE

The user selects 'R' for repeat. This time after each pressure is shown, (s)he presses the NO SCROLL key for a stop action, and a second, third, and fourth time to resume and stop the action as the room fills with colored dots.

<NO SCROLL>

Action stops.

<NO SCROLL>

Action resumes.

<NO SCROLL>

Action stops again.

<NO SCROLL>

Action resumes again.

The computer finishes and asks the user if (s)he is finished.

R_EPEAT
D_ONE

This time the user is done. They press 'D' and the computer displays the GET RESULTS MENU again.

F_REQUENCY RESPONSE
S_PATIAL DISTRIBUTION
M_AIN MENU

The user selects 'M' for the MAIN MENU. It is displayed again.

L_OAD NODAL FILE
G_ET OLD RESULTS
C_HANGE VARIABLES
S_UBMIT MODEL TO BE PROCESSED
N_ATURAL FREQUENCIES (HARD WALL)
W_RITE TO A FILE

E_XIT FROM ACOUSTIC SYSTEM

The user selects to change variables by pressing 'C'. The computer displays the CHANGE VARIABLES MENU.

F_REQUENCY DRIVING = 67.0
N_ODE DRIVING = 1
B_ETA ON EXISTING WALL SURFACES
 SURFACE NO. 1 ABSORPTION = 0.00500
 SURFACE NO. 2 ABSORPTION = 0.05000
P_REF = 2.0E-5
R_HO = 1.21
T_EMP = 24.5
Q_NOT = 1.7395E-3
I_NKJET = 0
M_AIN MENU

The user selects to change the driving frequency. (S)he selects 'F'. The computer asks for the new value.

ENTER NEW DRIVING FREQUENCY

The user types in a new value and presses the return key.

95

The CHANGE VARIABLES MENU is redisplayed with the updated information.

F_REQUENCY DRIVING = 95.0
N_ODE DRIVING = 1
B_ETA ON EXISTING WALL SURFACES
 SURFACE NO. 1 ABSORPTION = 0.00500
 SURFACE NO. 2 ABSORPTION = 0.05000
P_REF = 2.0E-5
R_HO = 1.21
T_EMP = 24.5
Q_NOT = 1.7395E-3
I_NKJET = 0
M_AIN MENU

The user selects to change the absorption on the soft wall, makig it the same as the other walls. (S)he presses 'B'. The computer responds:

ENTER SURFACE NO.

The user types in the surface number corresponding to the tilted facet of the trapezoidal room and presses the return key.

2

The computer asks for the new absorption value.

ENTER NEW ABSORPTION COEFFICIENT

The user enters a value that is the same for Surface No. 1 and presses the return key.

0.005

The CHANGE VARIABLES MENU is updated and redisplayed.

```
F_REQUENCY DRIVING = 95.0
N_ODE DRIVING = 1
B_ETA ON  EXISTING WALL SURFACES
 SURFACE NO. 1 ABSORPTION = 0.00500
 SURFACE NO. 2 ABSORPTION = 0.00500
P_REF = 2.0E-5
R_HO  = 1.21
T_EMP = 24.5
Q_NOT = 1.7395E-3
I_NKJET = C
M_AIN MENU
```

The user changes the display graphics to one more compatible for the inkjet plotter. (S)he presses 'I' for INKJET. The computer asks whether the runs are for the INKJET or not.

ARE RUNS FOR INKJET? Y_ES OR N_O

In future runs, the user may wish to use the inkjet plotter and type 'Y' at this point. However, this will turn the screen white during the next graphic display, making it difficult to read the future menus. It is recommended that the user use the current white on black background until (s)he is familiar with the system. The user types 'N' for this and the CHANGE VARIABLES MENU is redisplayed.

```
F_REQUENCY DRIVING = 95.0
N_ODE DRIVING = 1
B_ETA ON  EXISTING WALL SURFACES
 SURFACE NO. 1 ABSORPTION = 0.00500
 SURFACE NO. 2 ABSORPTION = 0.00500
P_REF = 2.0E-5
R_HO  = 1.21
```

T_EMP = 24.5
Q_NOT = 1.7395E-3
I_NKJET = 0
M_AIN MENU

The user returns to the MAIN MENU by typing 'M'.

L_OAD NODAL FILE
G_ET OLD RESULTS
C_HANGE VARIABLES
S_UBMIT MODEL TO BE PROCESSED
N_ATURAL FREQUENCIES (HARD WALL)
W_RITE TO A FILE
E_XIT FROM ACOUSTIC SYSTEM

Next, the user submits the model to be processed for the Acoustic Pressure by typing 'S'. The system responds with:

COMPUTING PRESSURES ...

Until it is finished. Then it displays the spatial distribution for the new case. When finished, it asks the user if (s)he wants an instant replay.

R_EPEAT
D_ONE

The user is finished and selects 'D'. The MAIN MENU reappears.

L_OAD NODAL FILE
G_ET OLD RESULTS
C_HANGE VARIABLES
S_UBMIT MODEL TO BE PROCESSED
N_ATURAL FREQUENCIES (HARD WALL)
W_RITE TO A FILE
E_XIT FROM ACOUSTIC SYSTEM

The user asks to see the natural frequencies by typing 'N'. The system responds with:

COMPUTING HARD WALL NATURAL FREQUENCIES ...

And follows that up with the first 30 natural frequencies.

MODE = 1, FREQUENCY = 68
MODE = 2, FREQUENCY = 96
MODE = 3, FREQUENCY = 117
. . . .

. . . .

MODE = 30, FREQUENCY = ...

And returns to the MAIN MENU.

L_OAD NODAL FILE
G_ET OLD RESULTS
C_HANGE VARIABLES
S_UBMIT MODEL TO BE PROCESSED
N_ATURAL FREQUENCIES (HARD WALL)
W_RITE TO A FILE
E_XIT FROM ACOUSTIC SYSTEM

The user selects to write this new information to a file for future recall. (S)he selects 'W' and studies the WRITE MENU.

CURRENT WRITE FILE IS UNINITIALIZED

S_PECIFY A NEW FILE TO WRITE TO
N_ODAL DATA - WRITE TO FILE
P_RESSURE DATA - WRITE TO FILE
H_ARD WALL NAT. FREQS. - WRITE TO FILE
M_AIN MENU

The user selects 's' to specify a file. The computer asks for a new file name.

ENTER FILE NAME

The user enters a new file name and presses the return key.

TRAP.DAT

The WRITE MENU reappears. The user writes the mesh data to the file by specifying 'N'. The system quickly verifies the action and redisplays the menu.

WRITING MESH DATA TO TRAP.DAT

CURRENT WRITE FILE IS TRAP.DAT

S_PECIFY A NEW FILE TO WRITE TO
N_ODAL DATA - WRITE TO FILE
P_RESSURE DATA - WRITE TO FILE
H_ARD WALL NAT. FREQS. - WRITE TO FILE
M_AIN MENU

The user writes the pressure data from the latest run to the file by pressing 'P'. Again the system verifies the action and rewrites the WRITE MENU.

WRITING PRESSURE DATA TO TRAP.DAT

CURRENT WRITE FILE IS TRAP.DAT

S_PECIFY A NEW FILE TO WRITE TO
N_ODAL DATA - WRITE TO FILE
P_RESSURE DATA - WRITE TO FILE
H_ARD WALL NAT. FREQS. - WRITE TO FILE
MAIN MENU

The user select 'M' to return to the MAIN MENU.

L_OAD NODAL FILE
G_ET OLD RESULTS
C_HANGE VARIABLES
S_UBMIT MODEL TO BE PROCESSED
N_ATURAL FREQUENCIES (HARD WALL)
W_RITE TO A FILE
E_XIT FROM ACOUSTIC SYSTEM

And 'E' to exit. The system wipes the screen clear of existing graphpics and leaves a message.

USER TERMINATION OF ACOUSTIC SYSTEM

VITA

Larry Emil Sabo was born in Phillipsburg, New Jersey, in 1959. He completed his Bachelor in Civil Engineering at the University of Delaware in 1981, where he concentrated in structural analysis and technical writing. After two years of employment with the Bureau of Bridges and Structures at New Jersey Department of Transportation, he returned to school to work on his Master of Science degree in Mechanical Engineering. He studied vibrations, acoustics, and computer aided engineering at Lehigh University in Bethlehem, Pennsylvania. He is currently employed by McDonnell Douglas Astronautics Space Programs in Huntington Beach, California where he is working on structural dynamics projects.